Predicting Friction System Performance with Symbolic Regression and Genetic Programming with Factor Variables

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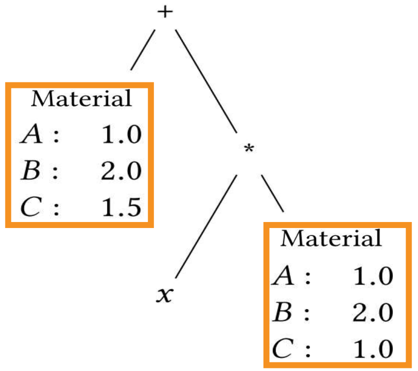
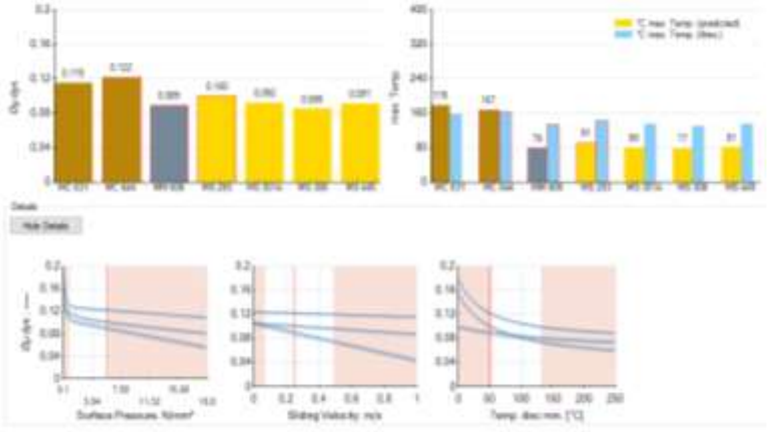
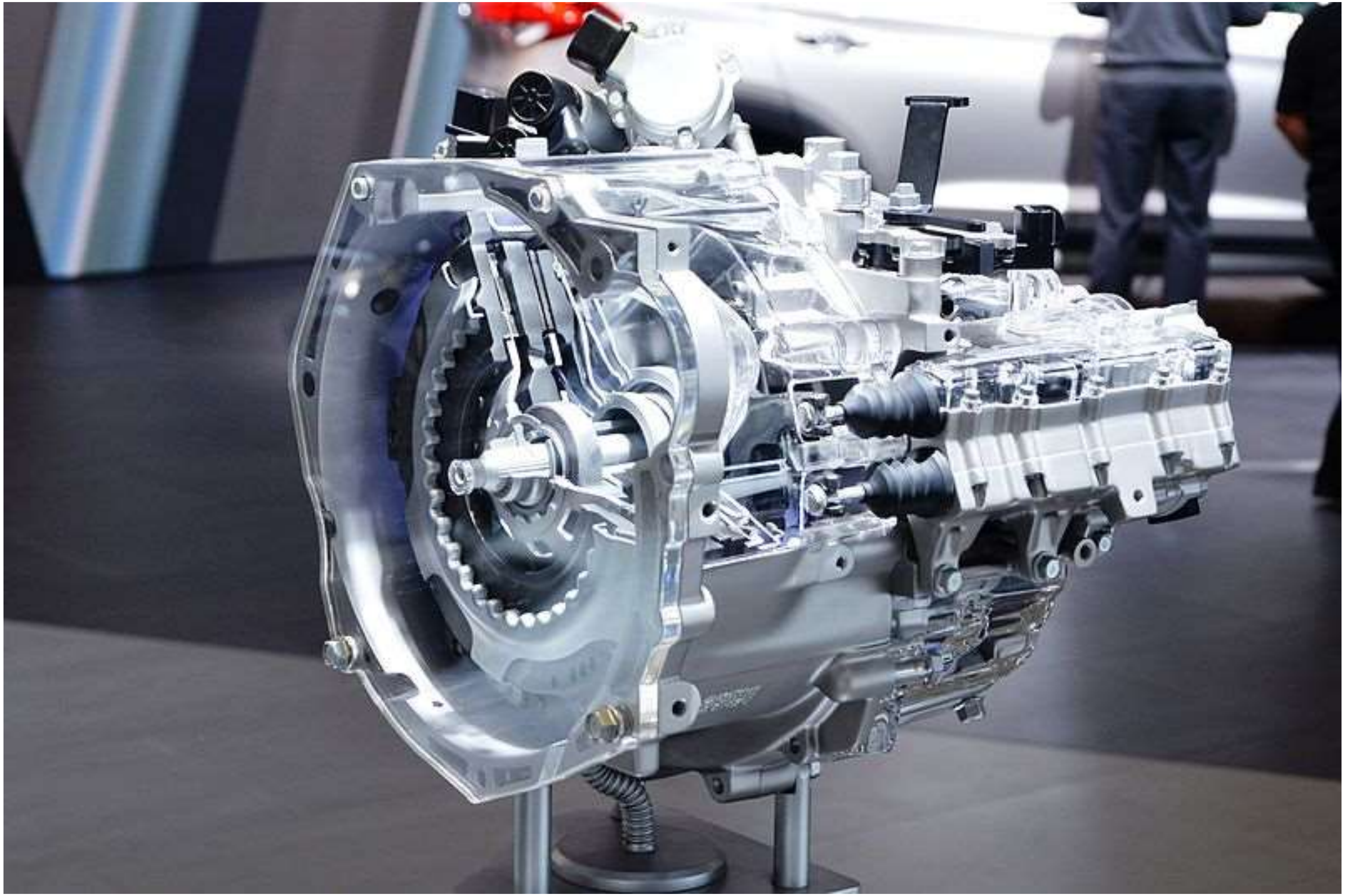
Abstract

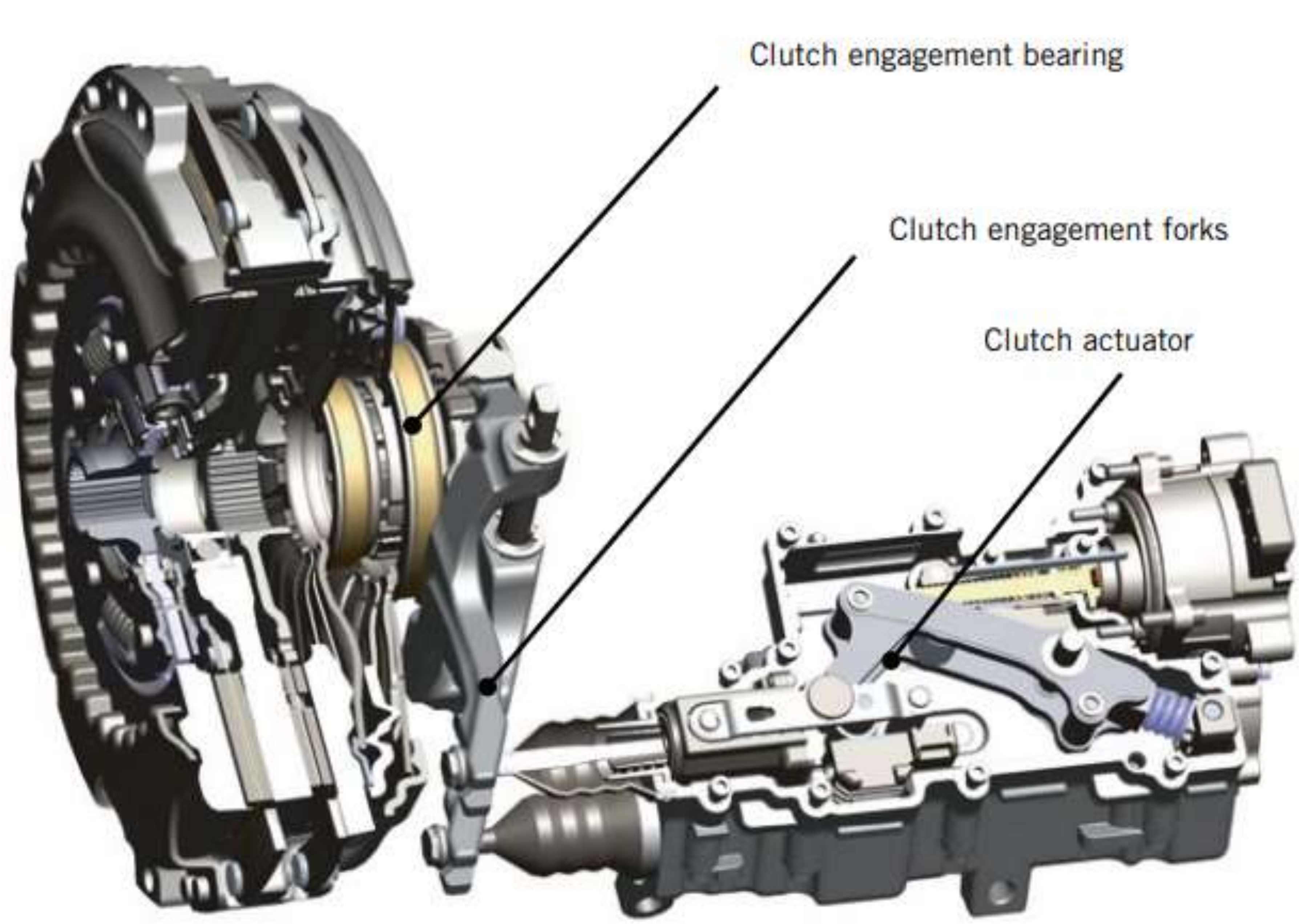
$$f(x, \text{Material}) = \underline{\text{Material}} \cdot x + \underline{\text{Material}}$$







Seven-speed-transmission Dual-clutch for various Hyundai-models. Photo taken at exhibition IAA 2015 in Frankfurt, Germany. © Spielvogel, Wikipedia



Source: Chang-Yeon Cho, Jeong-Heon Kam, Han-Ki Hong, Carsten Lövenich, *More Efficiency with the Dry Seven-speed Dual-clutch Transmission by Hyundai,* in ATZ Vol. 118, No. 06|2016, pp. 38 – 41, 2016
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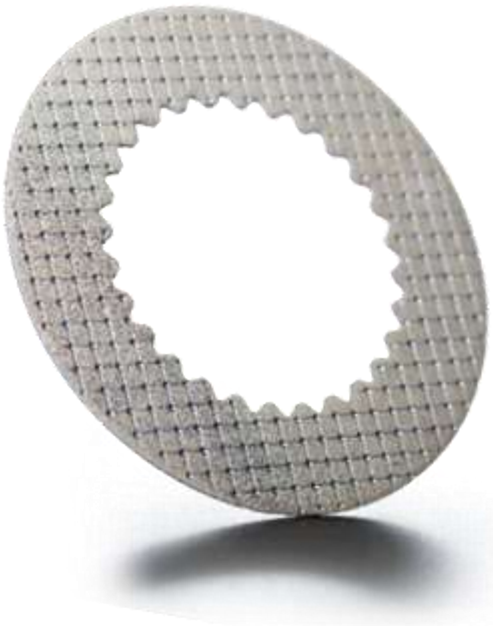
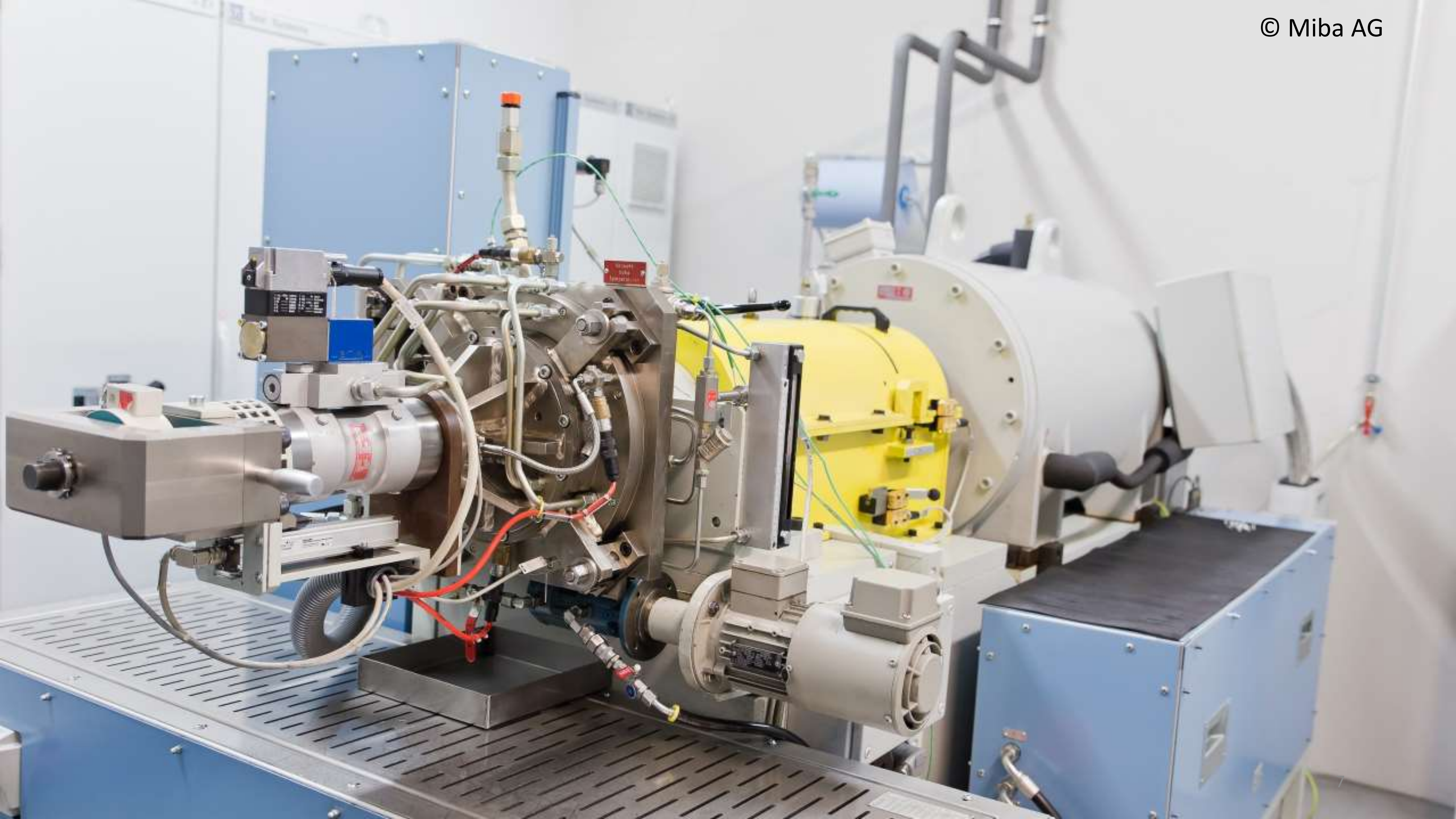
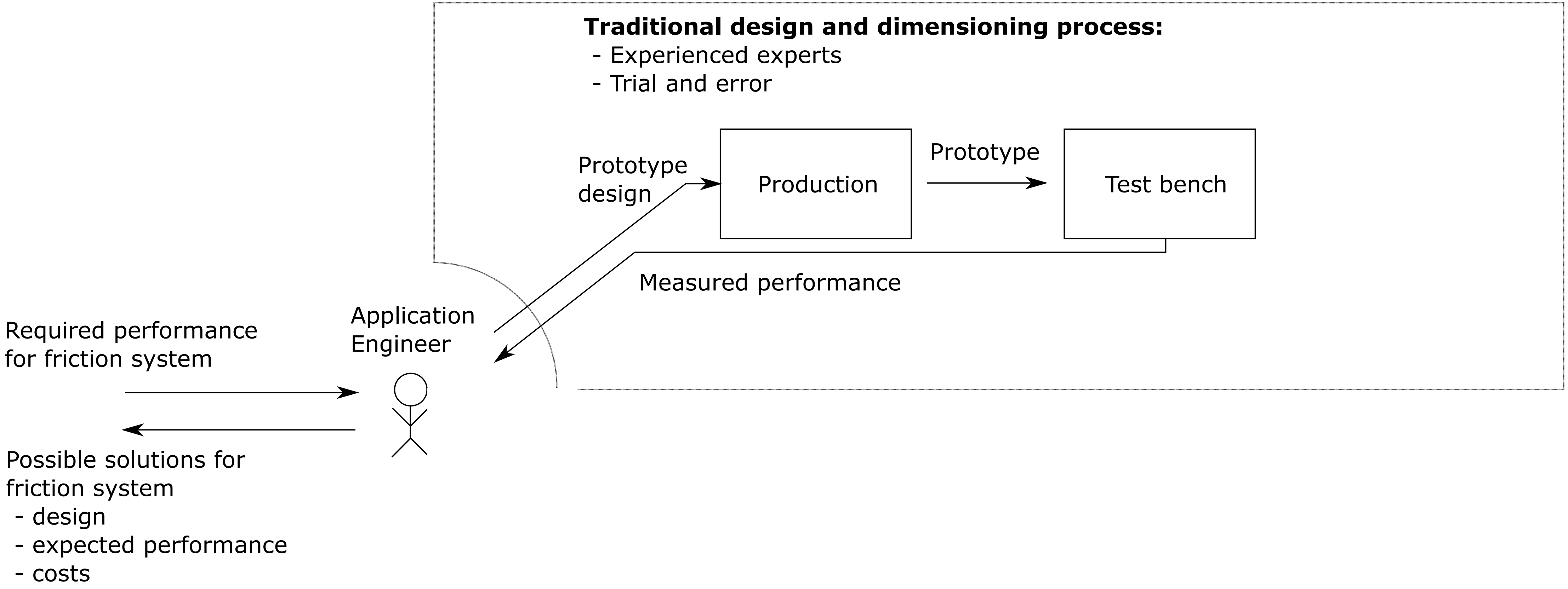
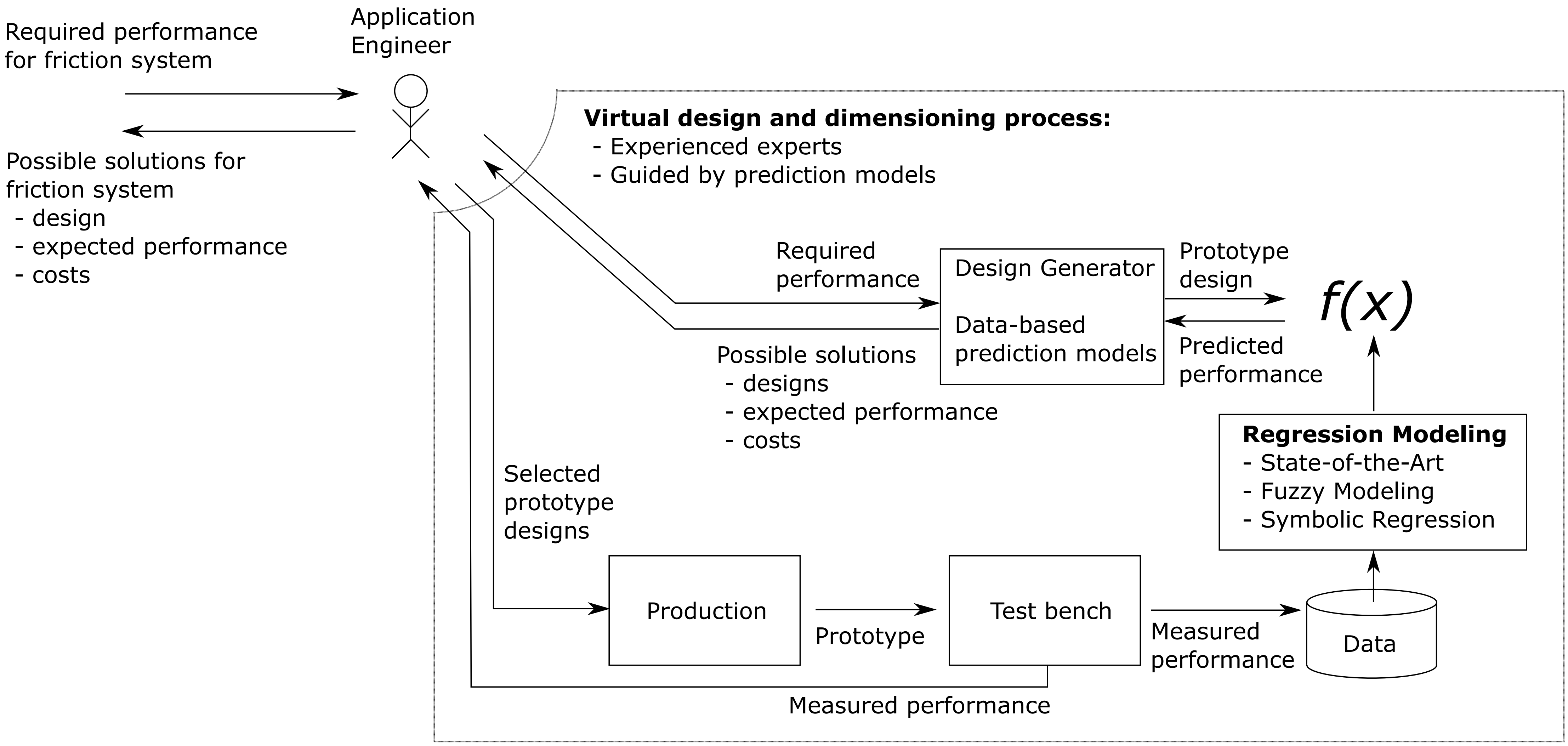
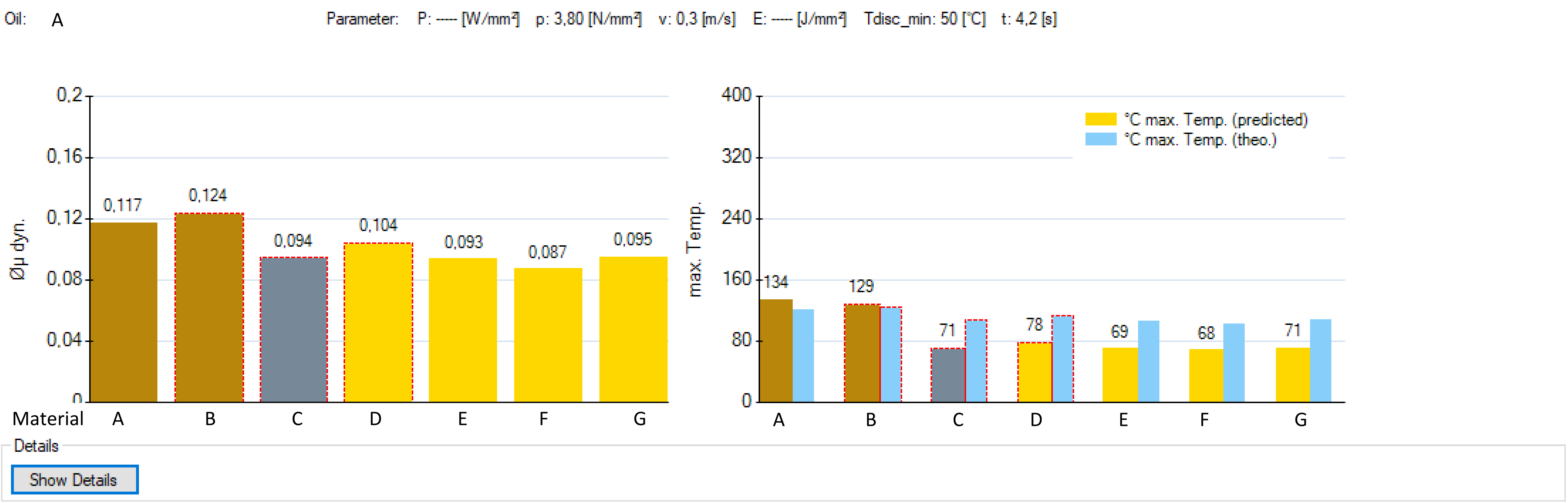





Traditional design and dimensioning process:
- Experienced experts
- Trial and error
Prototype design
Production
Prototype
Test bench
Measured performance
Application Engineer
Required performance for friction system
Possible solutions for friction system
- design
- expected performance
- costs


Required performance for friction system
Application Engineer
Possible solutions for friction system
- design
- expected performance
- costs
Virtual design and dimensioning process:
- Experienced experts
- Guided by prediction models
Required performance
Design Generator
Data-based prediction models
Prototype design
$f(x)$
Predicted performance
Possible solutions
- designs
- expected performance
- costs
Regression Modeling
- State-of-the-Art
- Fuzzy Modeling
- Symbolic Regression
Selected prototype designs
Production
Prototype
Test bench
Measured performance
Data
Measured performance


Oil: A
Parameter: P: ---- [W/mm²] p: 3,80 [N/mm²] v: 0,3 [m/s] E: ---- [J/mm²] Tdisc_min: 50 [°C] t: 4,2 [s]
Øμ dyn.
0,2
0,16
0,12
0,08
0,04
0
0,117
0,124
0,094
0,104
0,093
0,087
0,095
Material A B C D E F G
max. Temp.
400
320
240
160
80
0
°C max. Temp. (predicted)
°C max. Temp. (theo.)
134
129
71
78
69
68
71
A B C D E F G
Details
Show Details

Symbolic Regression

Given:

X … input matrix

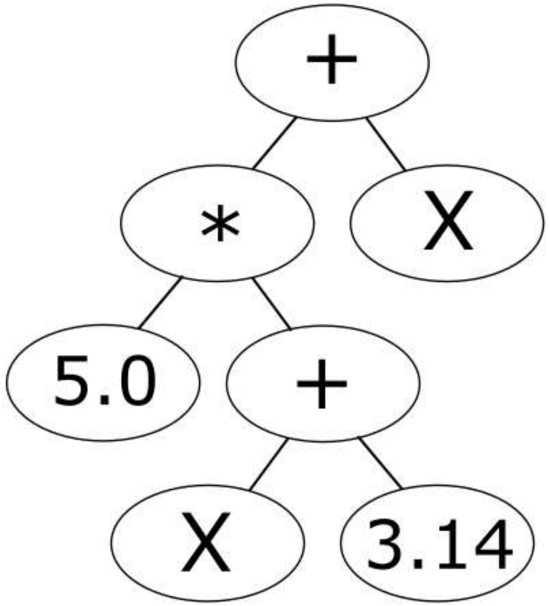
$\boldsymbol{y}$ … target vector

$L = G(\mathrm{Expr})$ … set of expressions defined via grammar G

$\mathrm{Obj}(X, \boldsymbol{y}, f)$ … objective function

Find: $$f^*(x) = \underset{f \in L}{\operatorname{argmin}} \, \mathrm{Obj}(\boldsymbol{y}, X, f)$$

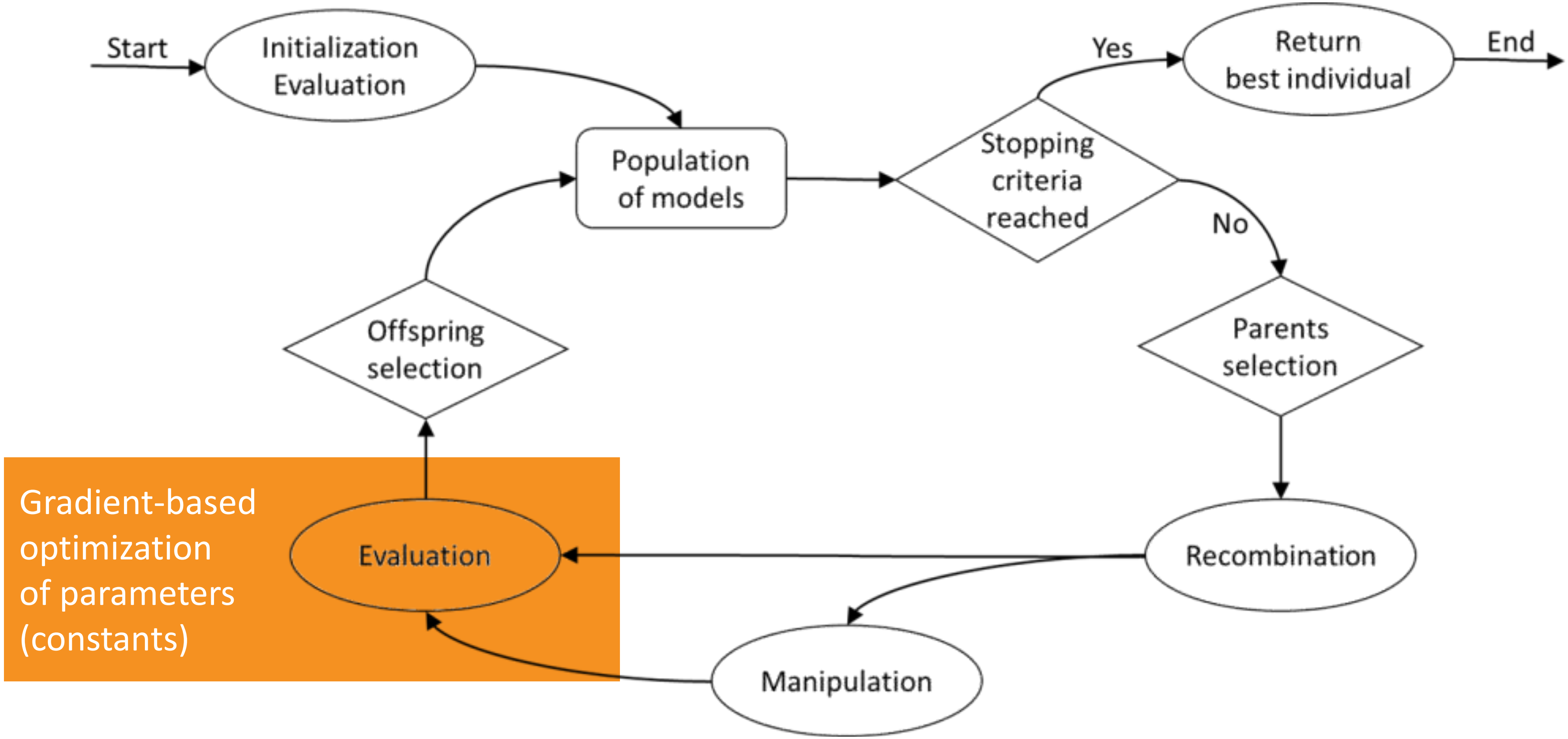
Grammar-guided Tree-based GP




```
G(Expr):
Expr = Term + Expr | Term
Term = Fact * Term | Fact
Fact = <const> | x
```

$$f(x) = 5 * (x + 3.14) + x$$

GP Workflow


Start
Initialization
Evaluation
Population
of models
Stopping
criteria
reached
Yes
Return
best individual
End
No
Parents
selection
Recombination
Manipulation
Evaluation
Offspring
selection
Gradient-based
optimization
of parameters
(constants)

Prediction of Friction System Performance

Target variables:

- friction coefficient
- change of temperature
- wear

Controlled variables:

- Numeric / continuous: temperature, pressure, sliding speed, inertia, …
- Nominal: friction material type, oil type, grooving type

First Approach: One-Hot-Encoding

Pressure, Speed, Energy, …	Material Type	Material Type A	Material Type B	Material Type C
	A	1	0	0
	A	1	0	0
	B	0	1	0
	C	0	0	1
	C	0	0	1
	C	0	0	1
	B	0	1	0

Difficulties with One-Hot-Encoding in GP

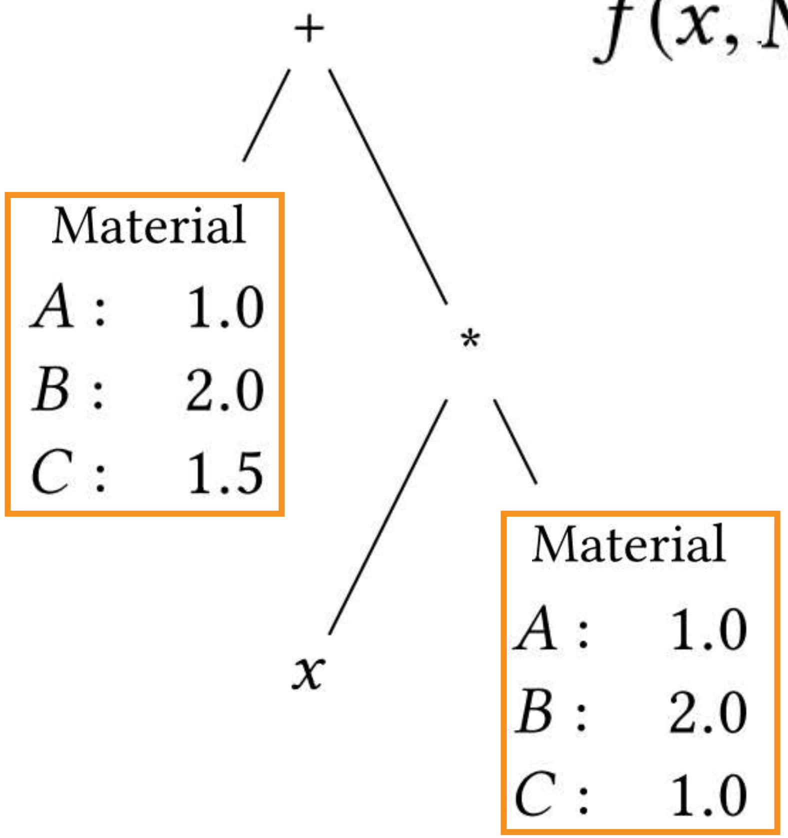
With one-hot encoding:

$$\widehat{\Delta T} = T + c_o \text{ cycles} + \mathrm{c}_1\, p + c_2 \mathrm{Mat}_A\, p + c_3 \mathrm{Mat}_B\, p + \frac{1}{p} c_4 \mathrm{Mat}_b + c_5 \mathrm{Mat}_\mathrm{A} + \exp(\mathrm{c}_6 \mathrm{Mat}_C)$$

With material-dependent “factors-variables”:

$$\widehat{\Delta T} = T + c_o \text{ cycles} + \mathrm{Mat}\, p + \mathrm{Mat}\frac{1}{p} + \mathrm{Mat}$$

The Novel GP Extension: Factor Variables



$$f(x, \text{Material}) = \underline{\text{Material}} \cdot x + \underline{\text{Material}}$$

x	Material	$f(x, \text{Material})$
3.0	A	4.0
2.0	B	6.0
1.0	C	2.5

Optimization of Factor Variables

We use gradient information and a Quasi-Newton method for optimizing all numeric parameters of symbolic regression models [Topchy and Punch, GECCO 2001] [Kommenda et al., GECCO 2013].

E.g. for optimizing the sum of squared errors:

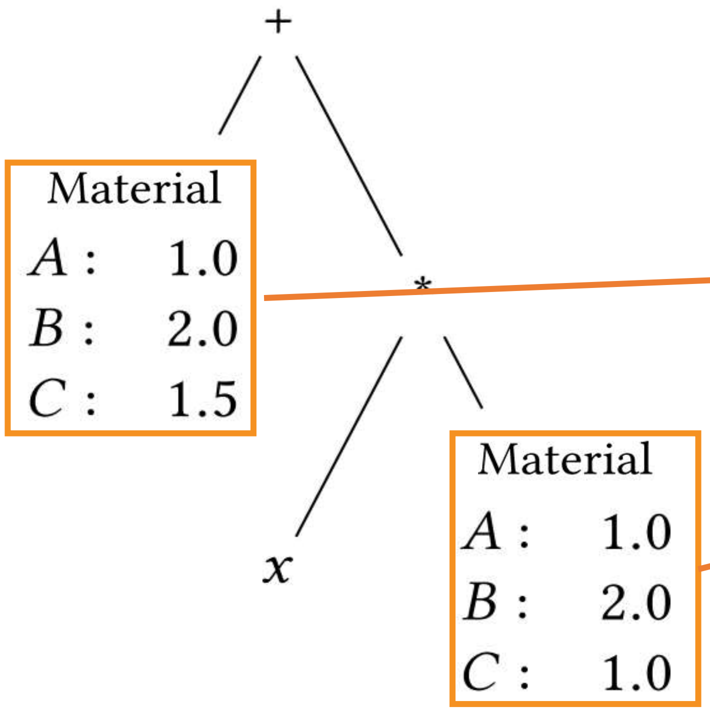
$$\underset{\beta}{\operatorname{argmin}} \sum_{i=0}^{m} (y_i - f(x_i, \beta))^2$$

The gradient of $f(x, \beta)$ over parameters β is useful and can be calculated easily

$$\nabla f = \left(\frac{\partial f}{\partial \beta_1}, \frac{\partial f}{\partial \beta_2}, \ldots, \frac{\partial f}{\partial \beta_n} \right)$$

Factor variables simply induce additional numeric parameters!

Optimization of Factor Variables



$$\theta = (\underbrace{1.0, 2.0, 1.5}, \underbrace{1.0, 2.0, 1.0})$$

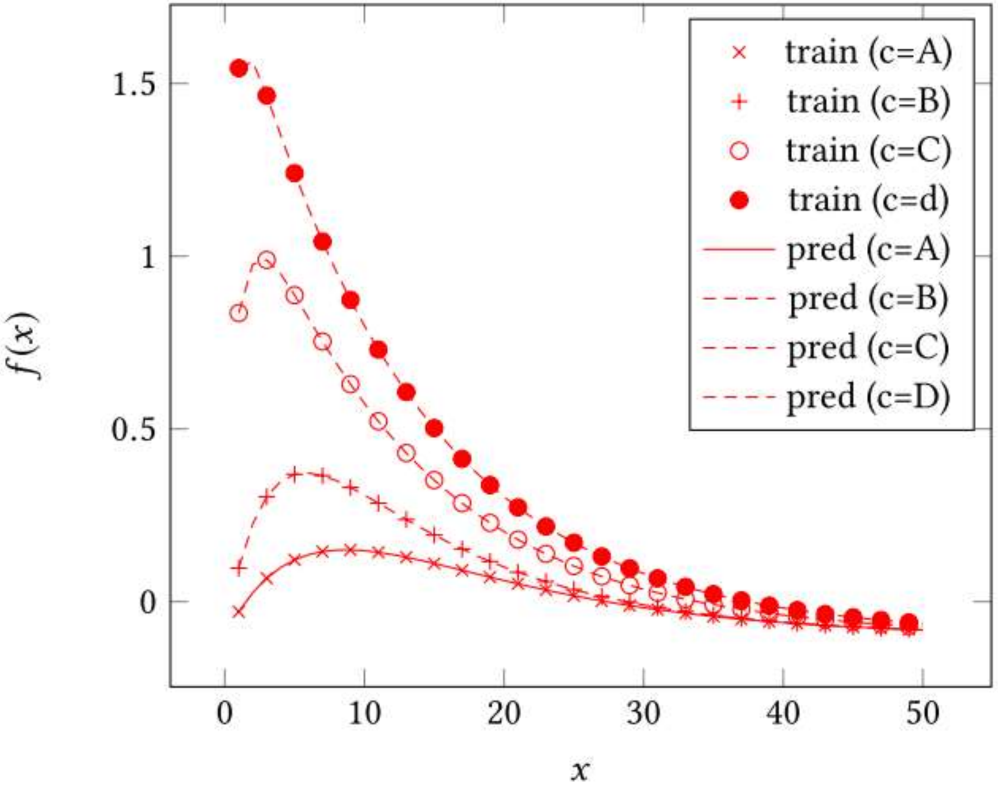
x	Material	$\frac{\delta f}{\delta \theta}(x, c)$
3.0	A	$(1, 0, 0, 3, 0, 0)$
2.0	B	$(0, 0, 1, 0, 0, 2)$
1.0	C	$(0, 1, 0, 0, 1, 0)$

- Gradient can be calculated easily using automatic differentiation with an effort factor of O(d)
- No numeric estimate of the gradient!

Does it work?

$$f(x) = \theta_{c,1} \exp(-0.08 \cdot x) - \exp(\theta_{c,2} \cdot x) - 0.1$$

c	θ_1	θ_2
A	1.0	−0.16
B	1.0	−0.32
C	1.5	−0.80
D	2.0	−1.60



It works!

Symbolic Regression Result

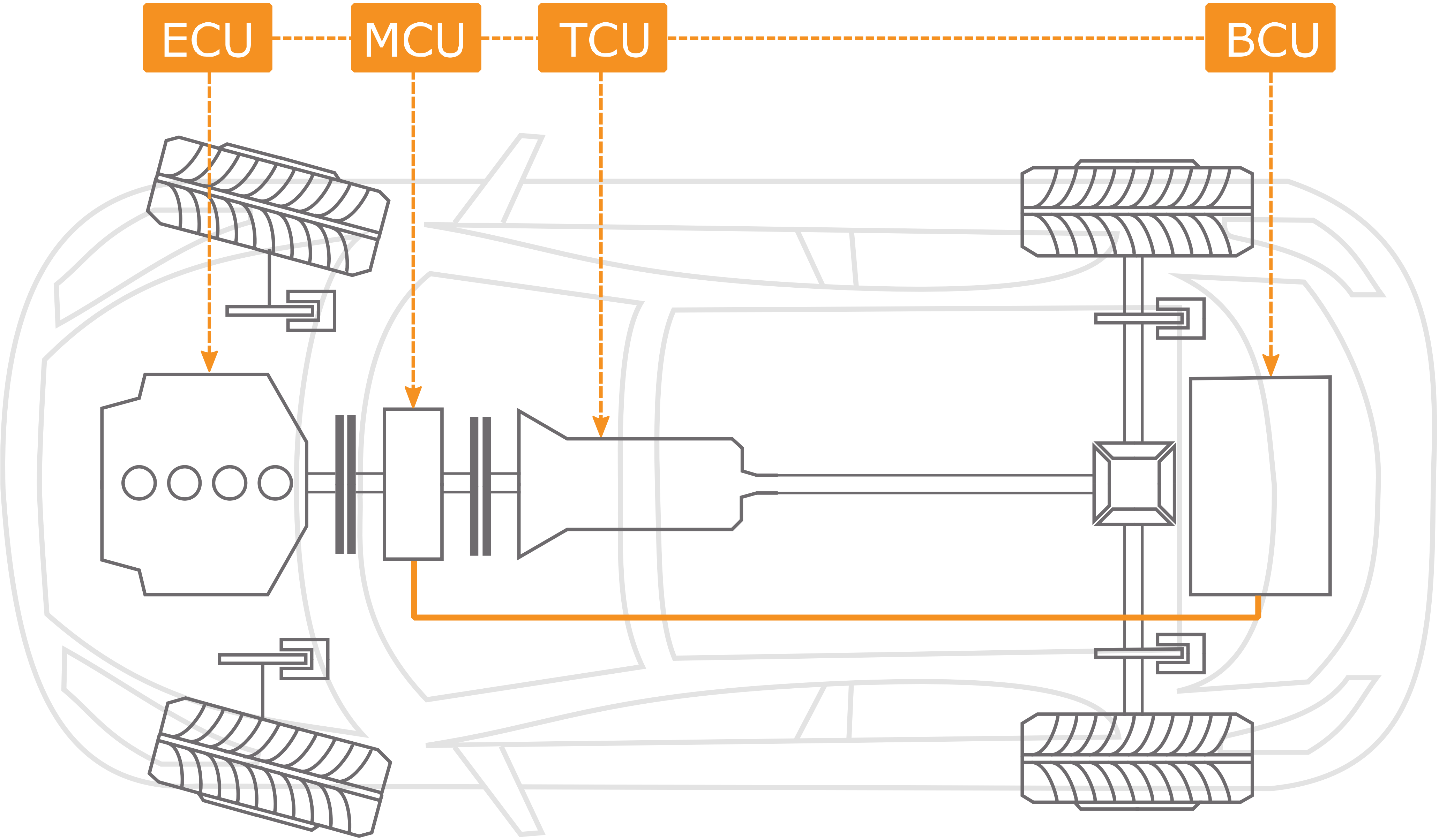
$$\hat{f}(x) = c_0 + c_1 \cdot \exp(x \cdot c_2) + c_3 \cdot \exp(c_4 \cdot x) \cdot \exp(c_5 \cdot x) + c_6$$

c_0	−0.25744	$c_{3,c=A}$	1.0
c_1	−1.0	$c_{3,c=B}$	1.0
$c_{2,c=A}$	−0.16	$c_{3,c=C}$	1.5
$c_{2,c=B}$	−0.32	$c_{3,c=D}$	2.0
$c_{2,c=C}$	−0.8	c_4	−0.034663
$c_{2,c=D}$	−1.6	c_5	−0.045337
		c_6	0.15744

Generating Function

$$f(x) = \theta_{c,1} \exp(-0.08 \cdot x) - \exp\left(\theta_{c,2} \cdot x\right) - 0.1$$

c	θ_1	θ_2
A	1.0	−0.16
B	1.0	−0.32
C	1.5	−0.80
D	2.0	−1.60


ECU
MCU
TCU
BCU

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Applied Soft Computing

Volume 69, August 2018, Pages 610-624

Using robust generalized fuzzy modeling and enhanced symbolic regression to model tribological systems

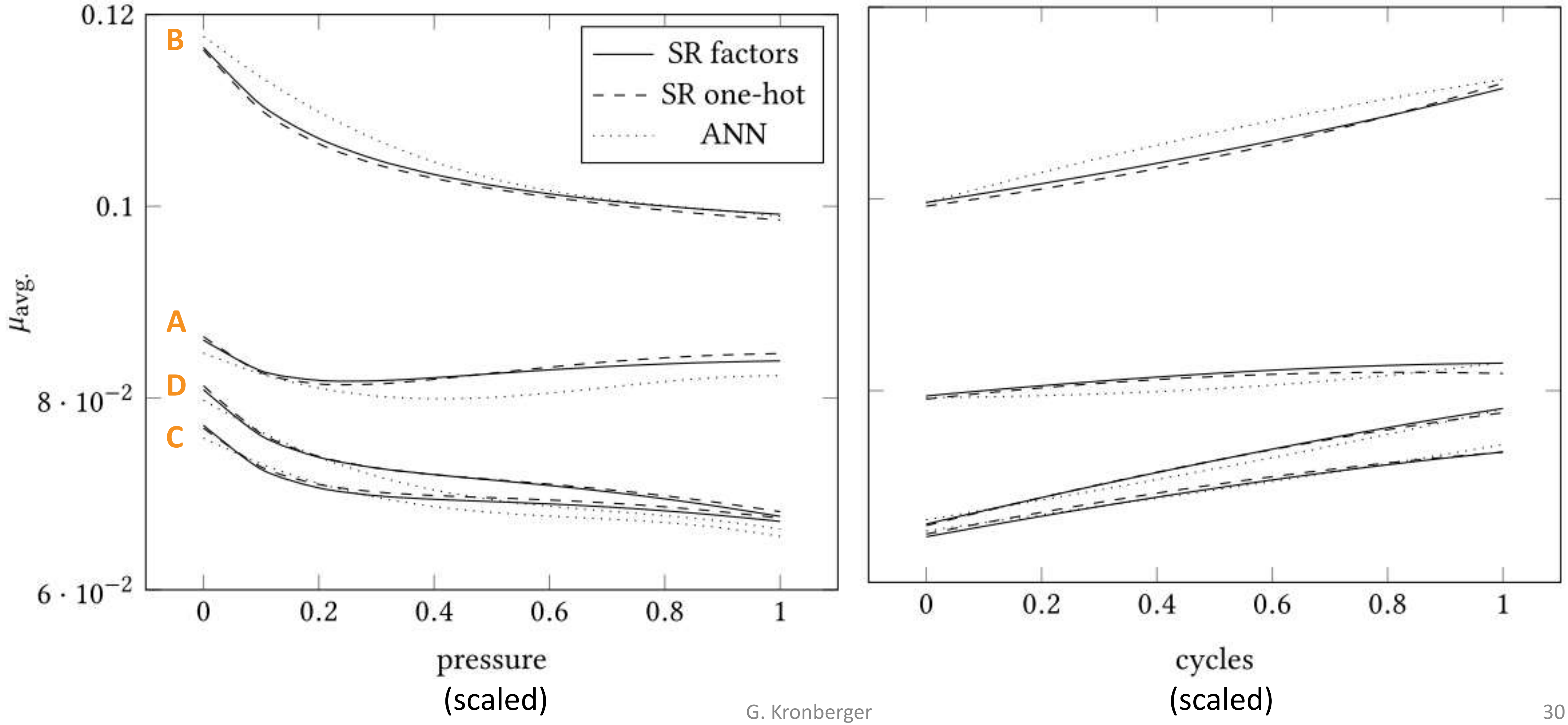
Gabriel Kronberger [a], Michael Kommenda [a], Edwin Lughofer [b], Susanne Saminger-Platz [b], Andreas Promberger [c], Falk Nickel [c], Stephan Winkler [a], Michael Affenzeller [a]

Public data set: http://dev.heuristiclab.com/AdditionalMaterial

Results for Predicting the Friction Coefficient

Model	average relative error
Linear regression	4.15 %
Artificial neural network	2.86 %
Random forest	3.08 %
Sym. reg. with one-hot-encoding	2.77 %
Sym. reg. with factor variables	2.84 %

Results for Friction Performance Prediction



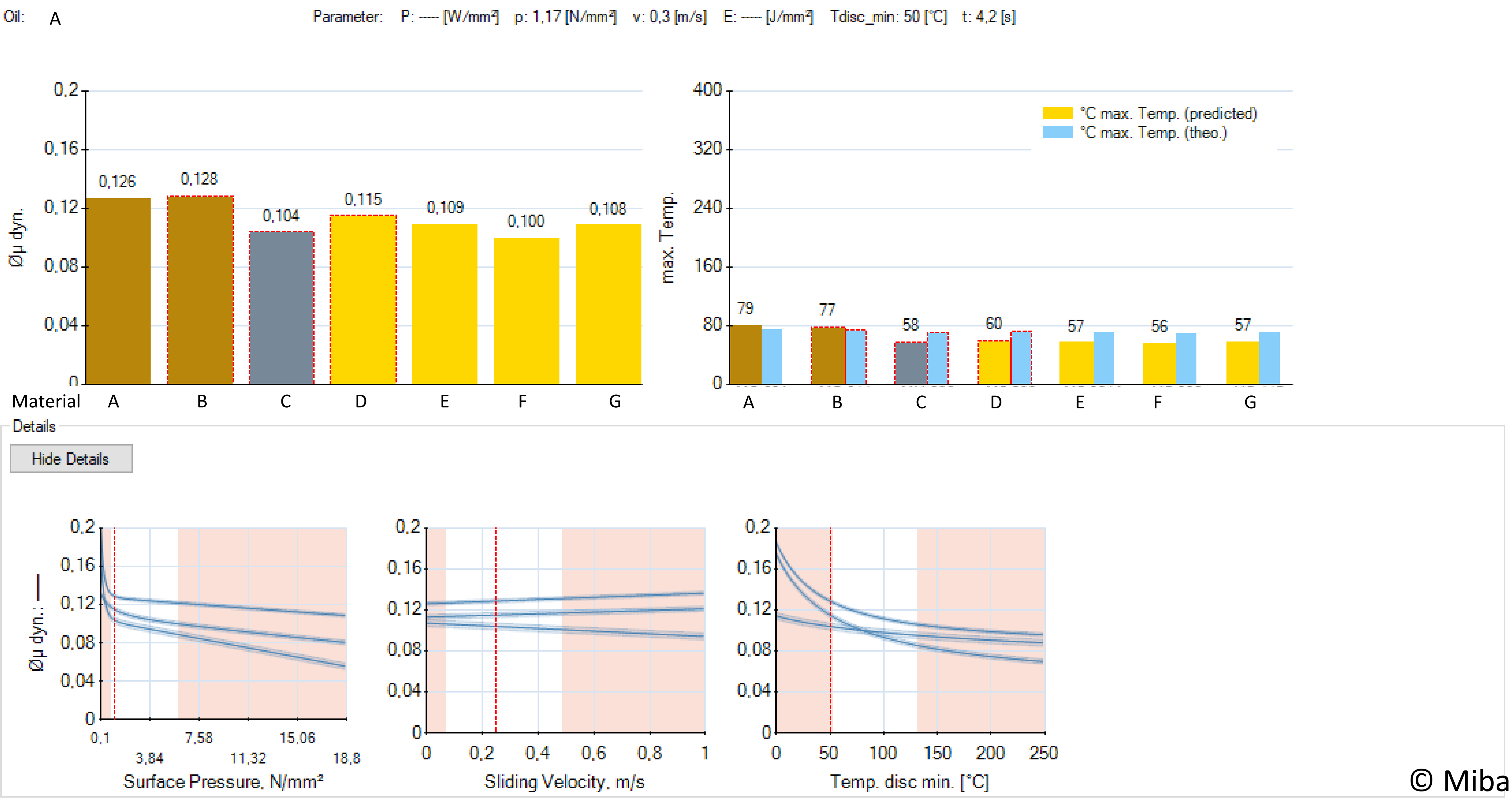
Results for Friction Performance Prediction

SR with one-hot encoding

$$\widehat{\mu_{avg}} = \frac{c_0 \cdot \text{cycles} + c_1 \cdot p + c_2 \cdot \underline{\text{M=b}} + \exp(c_3 \cdot \underline{\text{M=c}}) \cdot c_4}{\exp(c_5 \cdot \underline{\text{M=d}} + c_6 \cdot \text{cycles} + c_7 \cdot p) \cdot c_8} + \exp(c_9 \cdot \underline{\text{M=d}} + c_{10} \cdot p) \cdot c_{11} + \frac{c_{12} \cdot \text{cycles} + c_{13} \cdot \underline{\text{M=b}} + c_{14} \cdot p + c_{15} \cdot \underline{\text{M=c}} + c_{16} \cdot \underline{\text{M=d}}}{\exp(c_{17} \cdot \underline{\text{M=b}} + c_{18} \cdot p) \cdot (c_{19} \cdot p + c_{20} \cdot \text{cycles}) + c_{21}} + c_{22}$$

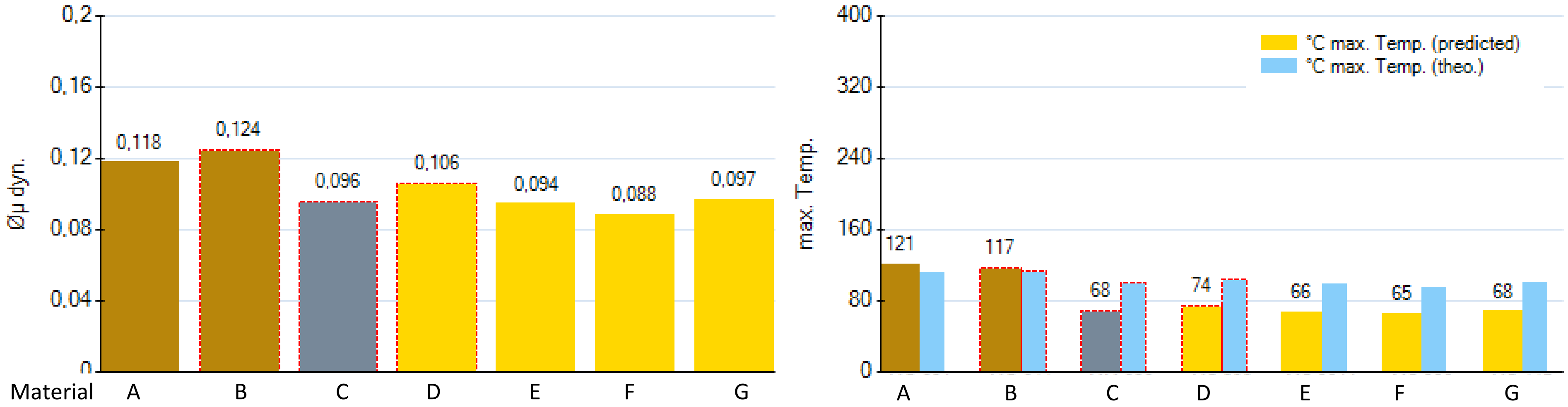
SR with factor variables

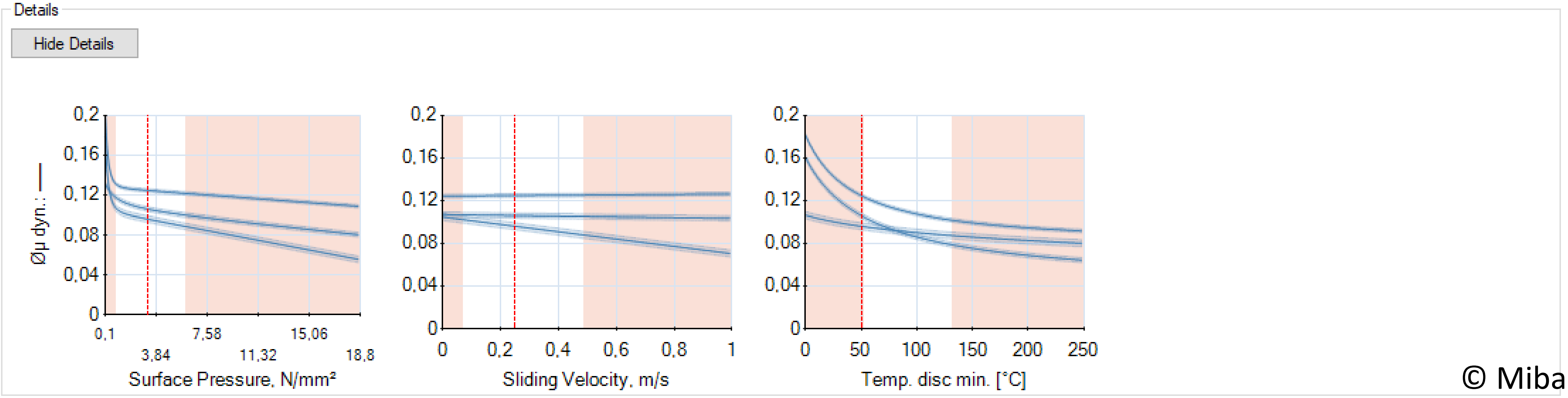
$$\widehat{\mu_{avg}} = \left(c_0 \cdot \text{cycles} + \underline{c_{1,M}} \cdot \exp(c_2 \cdot \text{cycles} + c_3 \cdot p) + \underline{c_{4,M}} \cdot (c_5 \cdot p + \exp(c_6 \cdot p) + c_7)\right) \cdot \frac{1}{c_8 \cdot p + \underline{c_{9,M}} + c_{10}} + c_{11}$$

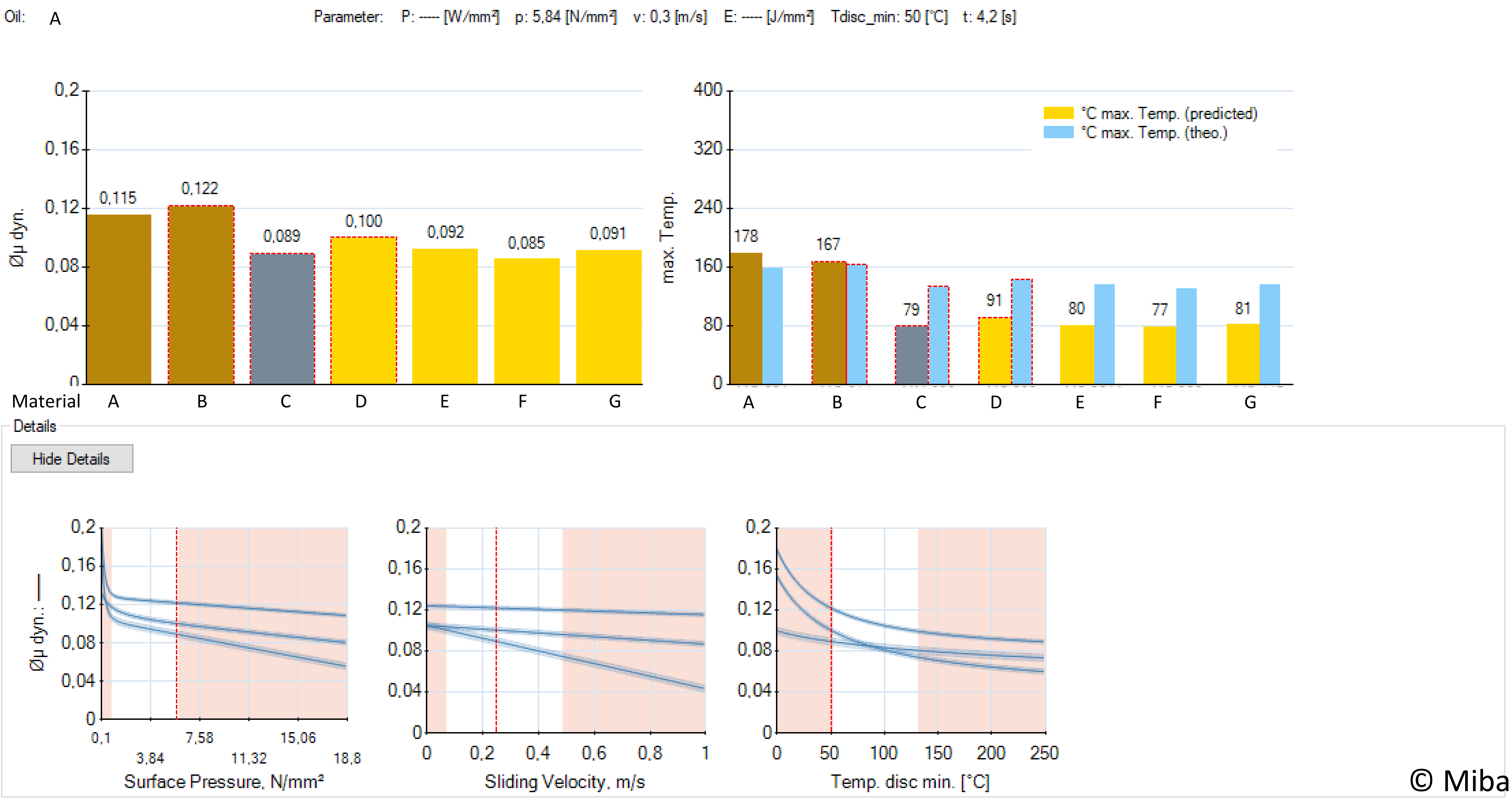

Oil: A
Parameter: P: ---- [W/mm²] p: 1,17 [N/mm²] v: 0,3 [m/s] E: ---- [J/mm²] Tdisc_min: 50 [°C] t: 4,2 [s]
Øμ dyn.
0,2
0,16
0,12
0,08
0,04
0
0,126
0,128
0,104
0,115
0,109
0,100
0,108
Material
A
B
C
D
E
F
G
max. Temp.
400
320
240
160
80
0
°C max. Temp. (predicted)
°C max. Temp. (theo.)
79
77
58
60
57
56
57
Details
Hide Details
Øμ dyn.:
Surface Pressure, N/mm²
0,1
3,84
7,58
11,32
15,06
18,8
Sliding Velocity, m/s
0
0,2
0,4
0,6
0,8
1
Temp. disc min. [°C]
0
50
100
150
200
250

Oil: A

Parameter: P: ---- [W/mm²] p: 3,21 [N/mm²] v: 0,3 [m/s] E: ---- [J/mm²] Tdisc_min: 50 [°C] t: 4,2 [s]






Oil: A
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Øμ dyn.
0,2
0,16
0,12
0,08
0,04
0
0,115
0,122
0,089
0,100
0,092
0,085
0,091
Material
A
B
C
D
E
F
G
max. Temp.
400
320
240
160
80
0
°C max. Temp. (predicted)
°C max. Temp. (theo.)
178
167
79
91
80
77
81
Details
Hide Details
Øμ dyn.:
Surface Pressure, N/mm²
0,1
3,84
7,58
11,32
15,06
18,8
Sliding Velocity, m/s
0
0,2
0,4
0,6
0,8
1
Temp. disc min. [°C]
0
50
100
150
200
250

Summary

- Introduced factor variables for genetic programming
- Described gradient-based optimization of factor variables
- Successfully applied to friction system prediction

Factor variables are useful for modeling systems with nominal variables which change the response of the system but not the fundamental mechanics of the system.

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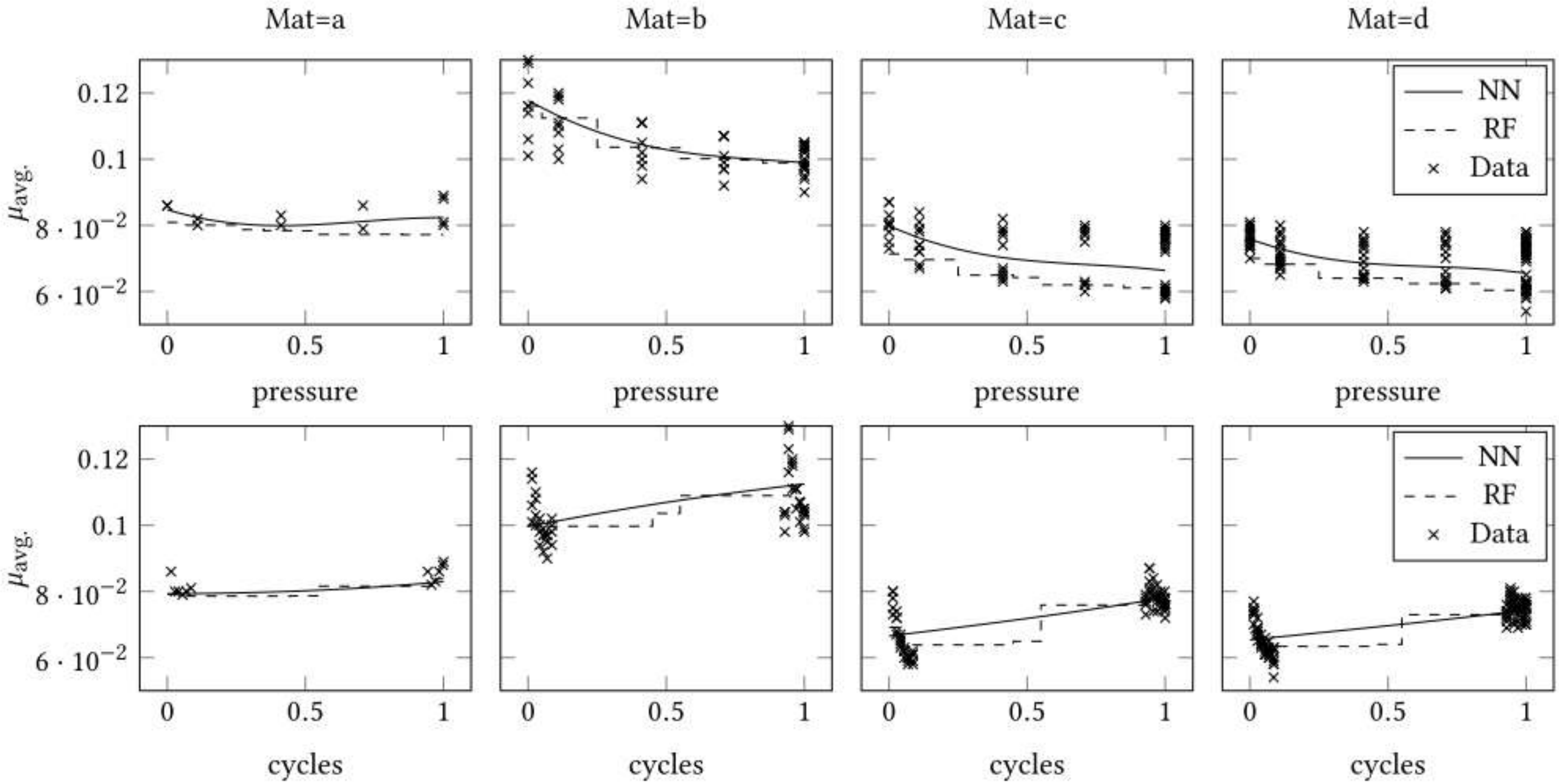
SymReg JOSEF RESSEL CENTER FOR SYMBOLIC REGRESSION


Mat=a
Mat=b
Mat=c
Mat=d
NN
RF
Data
$\mu_{avg.}$
0.12
0.1
$8 \cdot 10^{-2}$
$6 \cdot 10^{-2}$
0
0.5
1
pressure
cycles