

Predicting Friction System Performance with Symbolic Regression and Genetic Programming with Factor Variables

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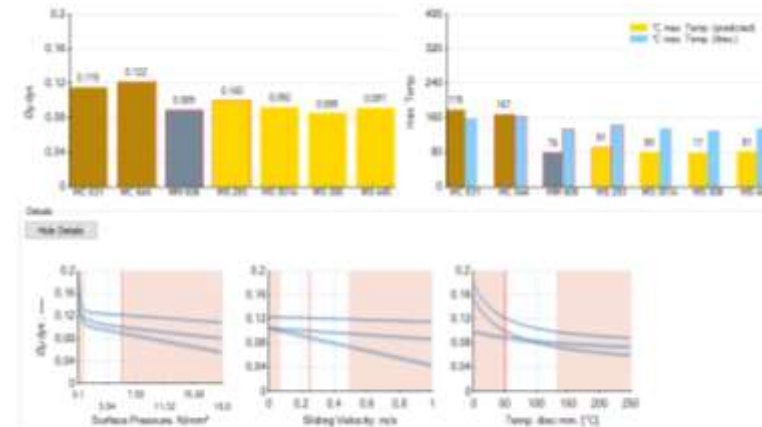
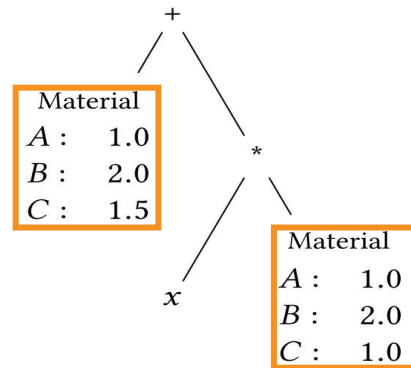
SymReg

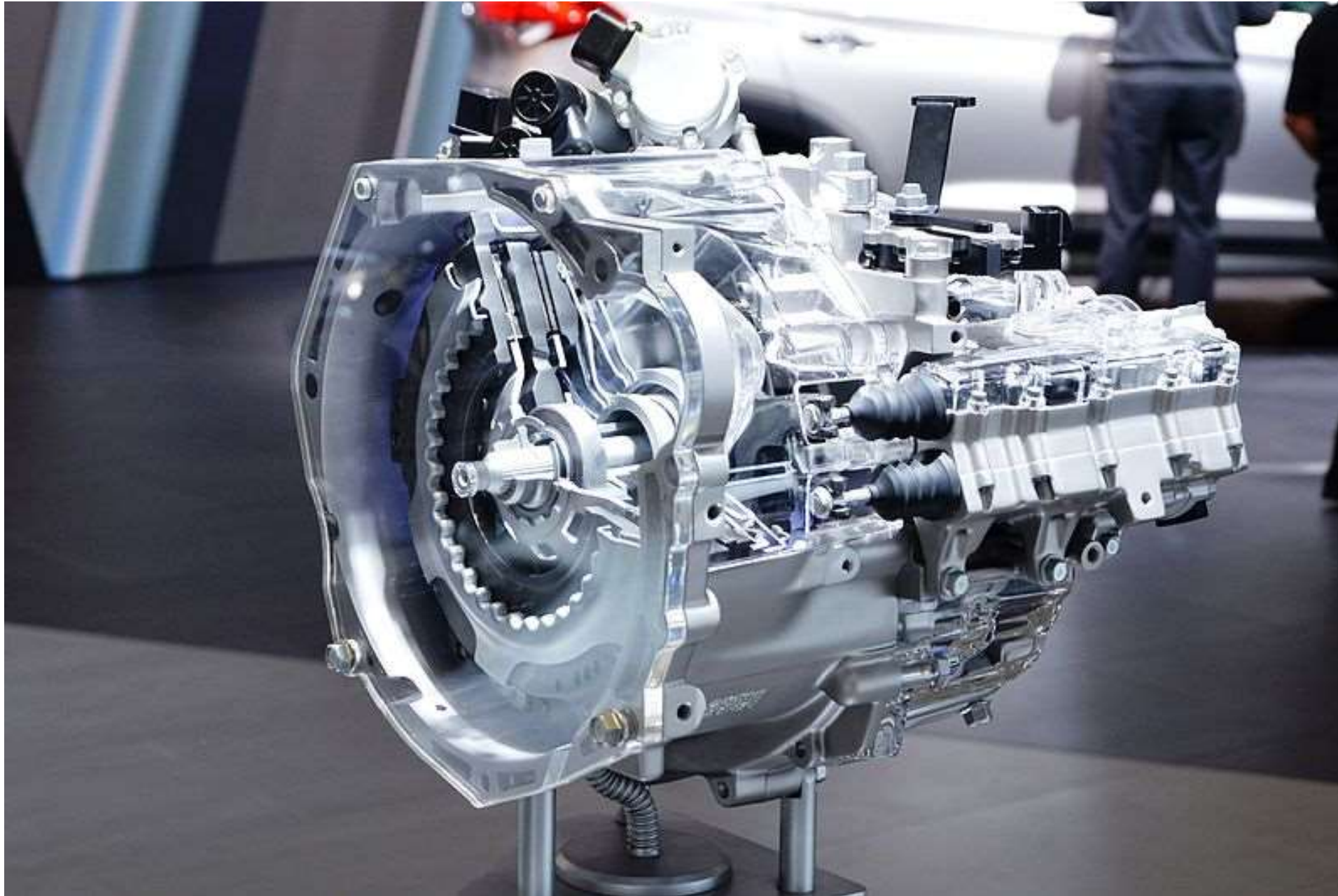
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Abstract

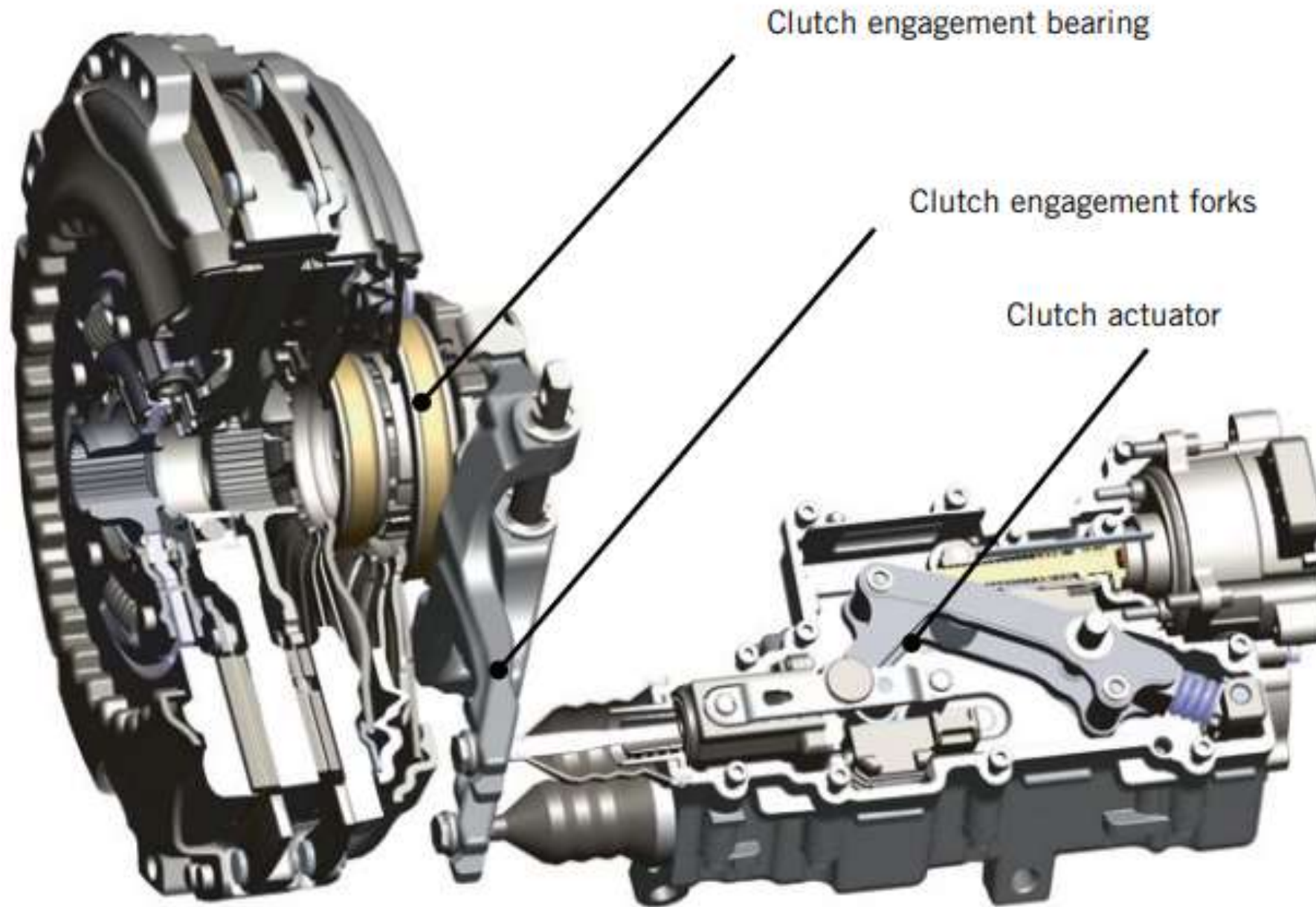


$$f(x, \text{Material}) = \underline{\text{Material}} \cdot x + \underline{\text{Material}}$$



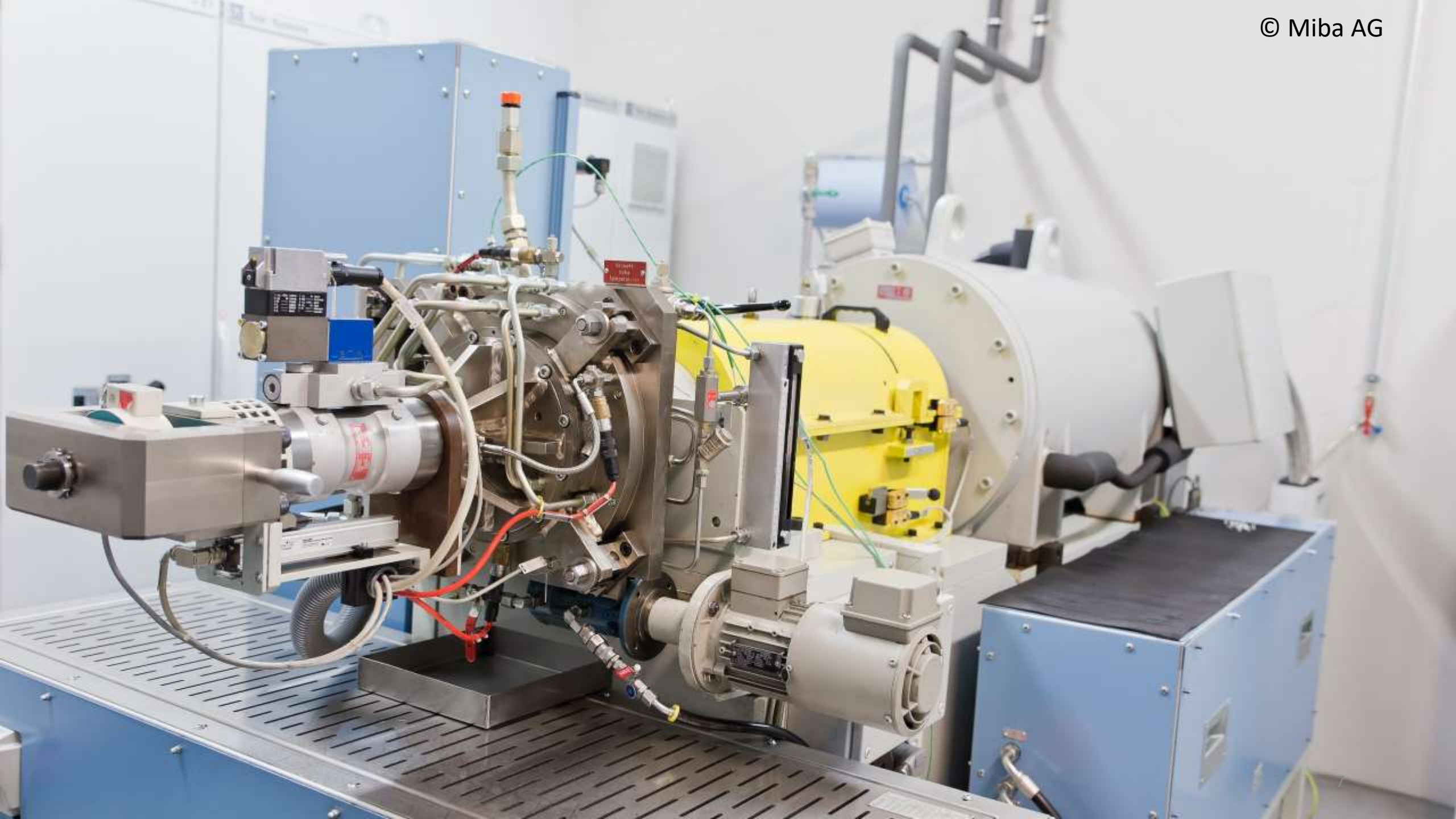


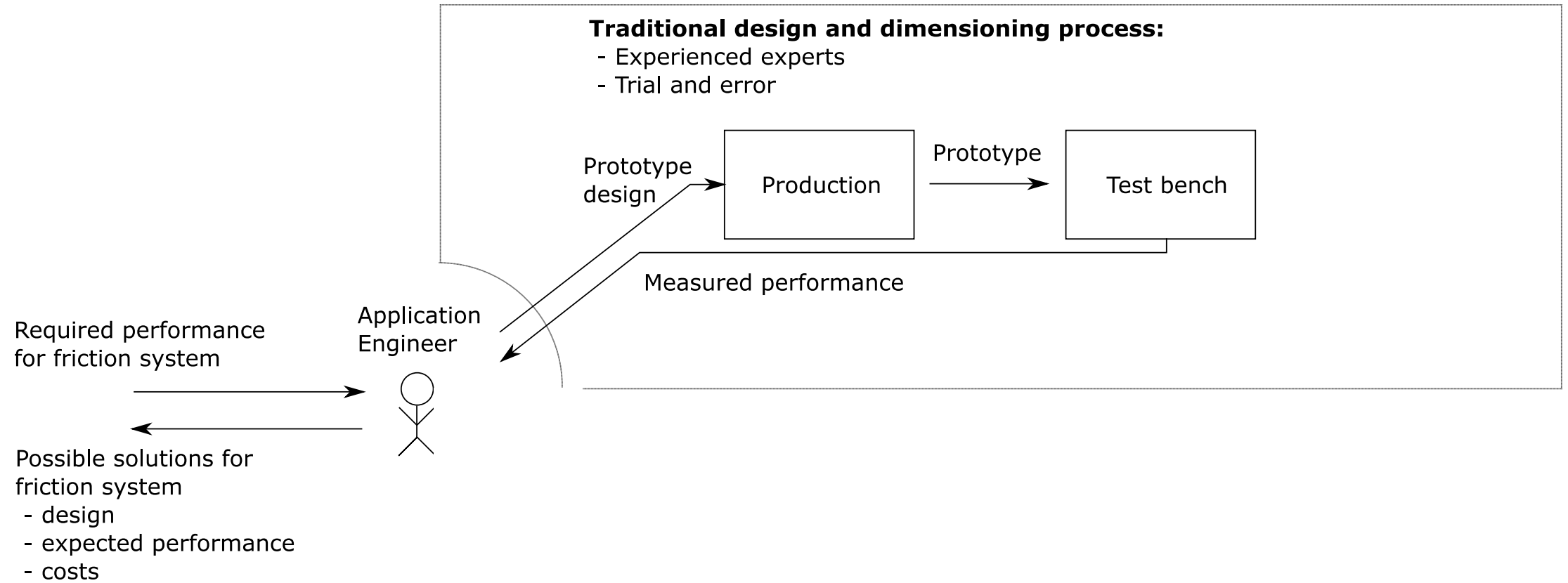
Seven-speed-
transmission Dual-
clutch for various
Hyundai-models.
Photo taken at
exhibition IAA 2015 in
Frankfurt, Germany.
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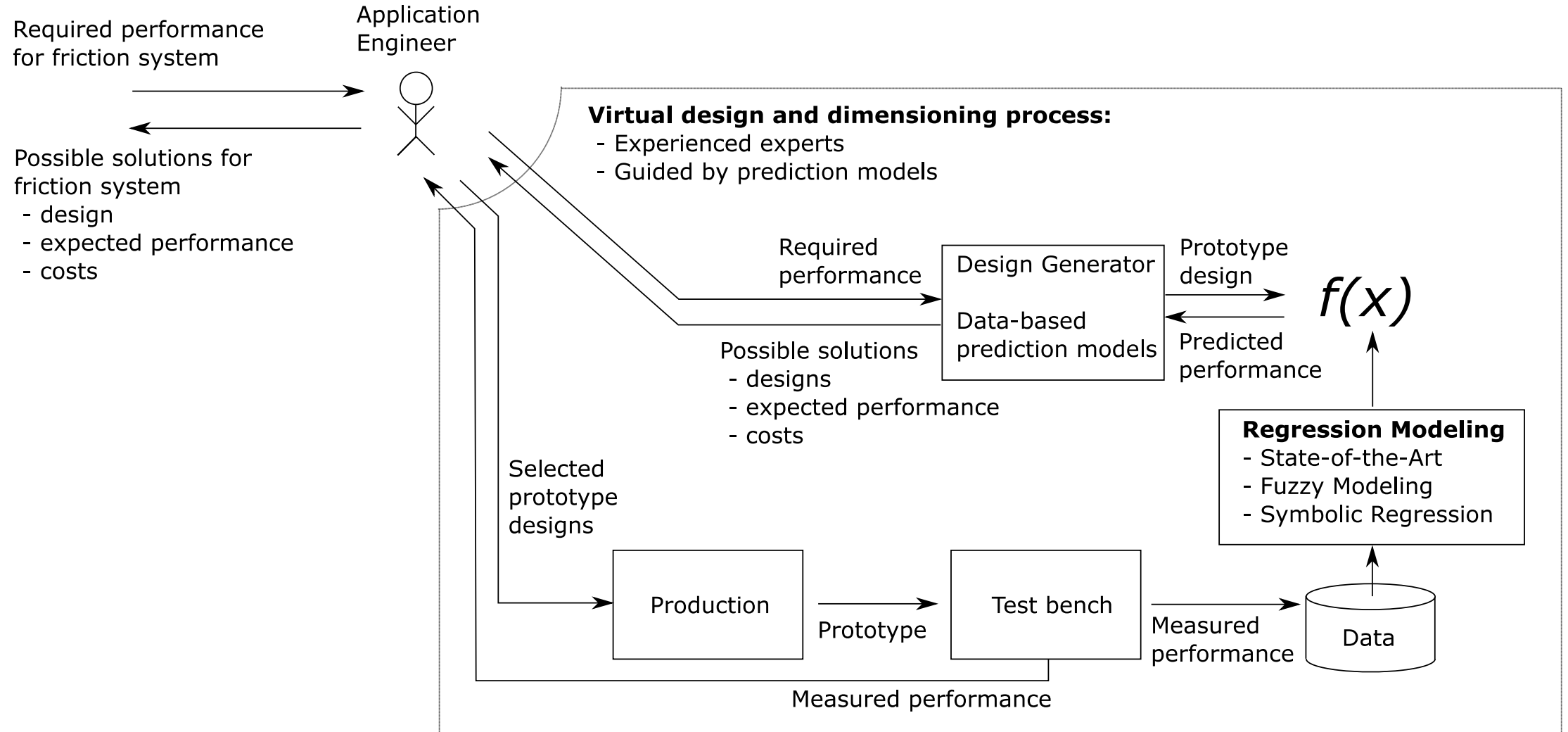


Source: Chang-Yeon Cho,
Jeong-Heon Kam, Han-Ki
Hong, Carsten Lövenich, *More
Efficiency with the Dry Seven-
speed Dual-clutch
Transmission by Hyundai*, in
ATZ Vol. 118, No. 06|2016, pp.
38 – 41, 2016
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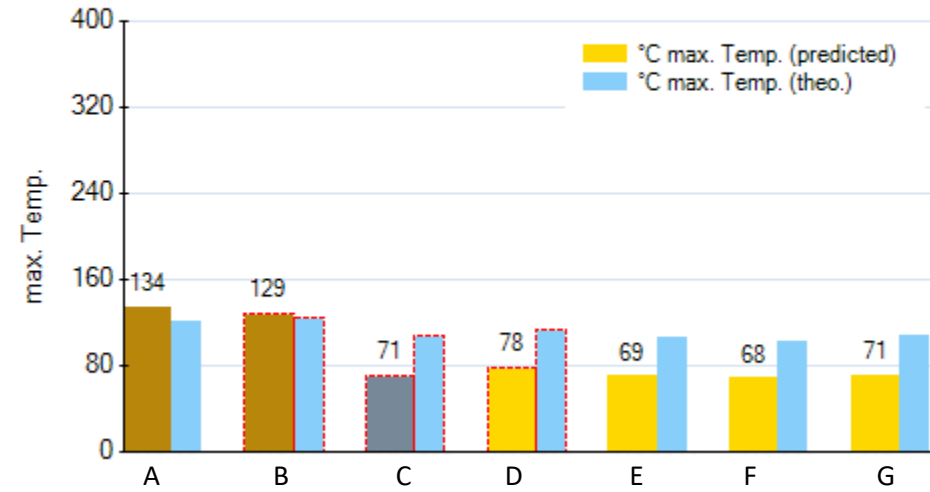
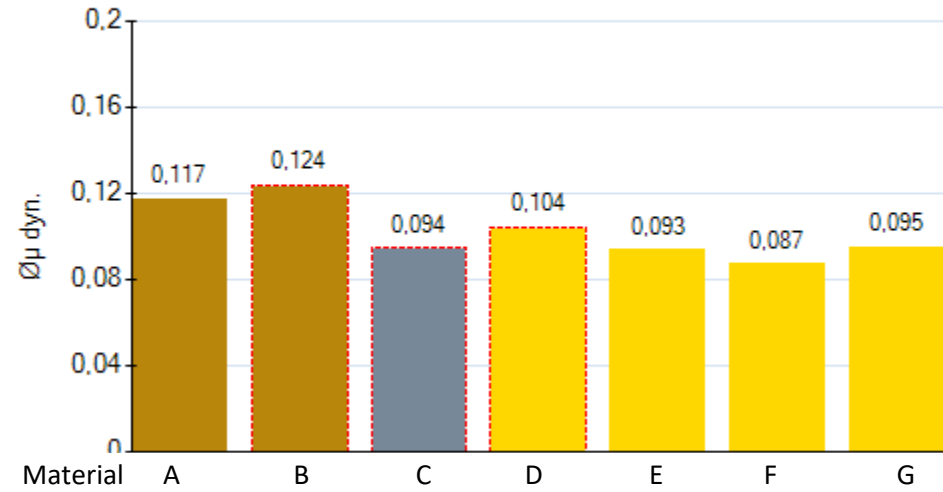






Oil: A

Parameter: P: — [W/mm²] p: 3,80 [N/mm²] v: 0,3 [m/s] E: — [J/mm²] Tdisc_min: 50 [°C] t: 4,2 [s]



Details

Show Details

Symbolic Regression

Given:

X ... input matrix

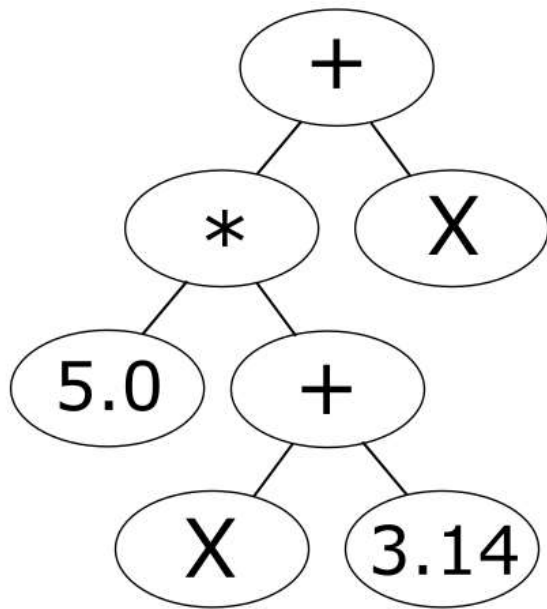
$\mathbf{y}$... target vector

$L = G(\text{Expr})$... set of expressions defined via grammar G

$\text{Obj}(X, \mathbf{y}, f)$... objective function

Find: $f^*(x) = \underset{f \in L}{\operatorname{argmin}} \text{Obj}(\mathbf{y}, X, f)$

Grammar-guided Tree-based GP



$$f(x) = 5 * (x + 3.14) + x$$

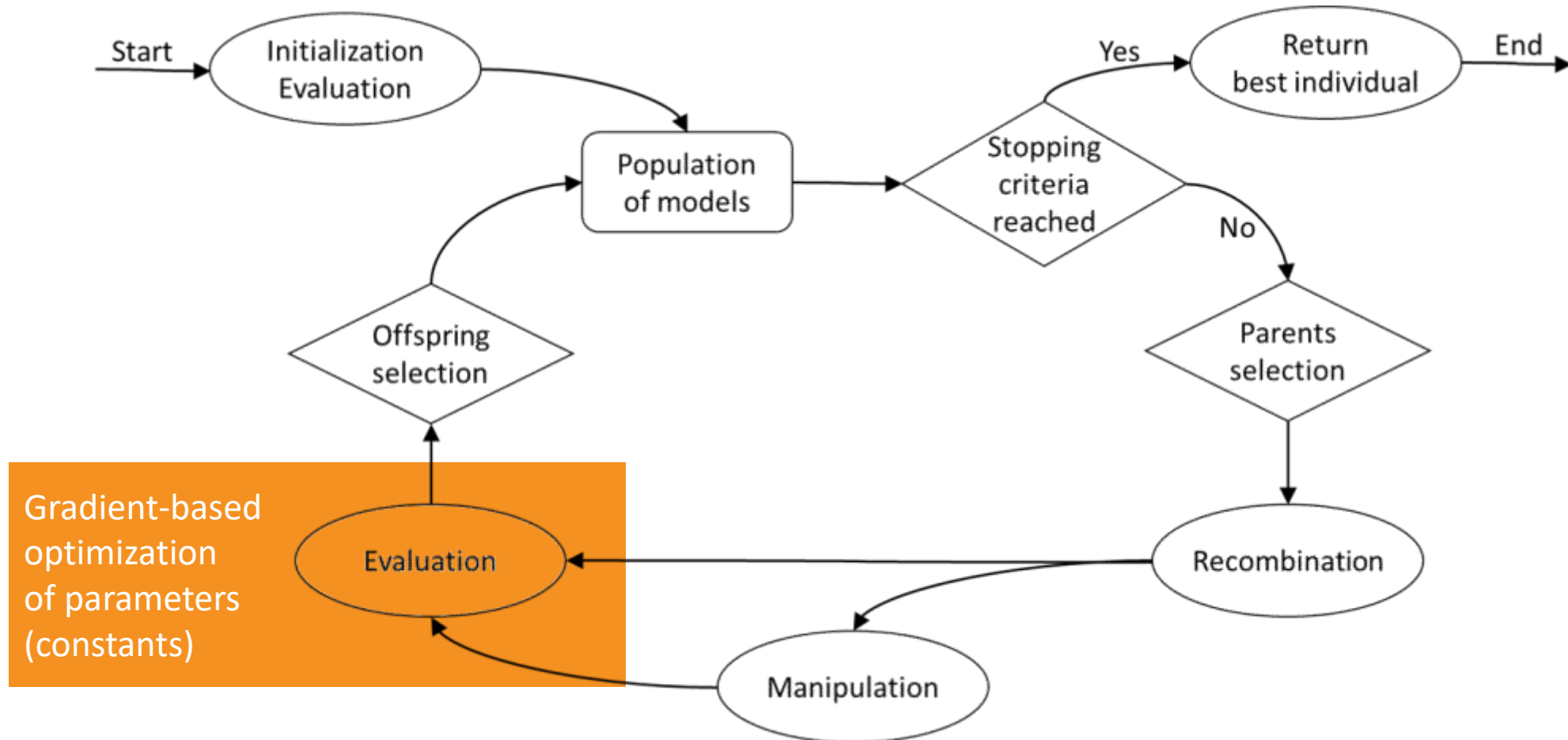
$G(\text{Expr}) :$

$\text{Expr} = \text{Term} + \text{Expr} \mid \text{Term}$

$\text{Term} = \text{Fact} * \text{Term} \mid \text{Fact}$

$\text{Fact} = \langle \text{const} \rangle \mid x$

GP Workflow



Prediction of Friction System Performance

Target variables:

- friction coefficient
- change of temperature
- wear

Controlled variables:

- Numeric / continuous: temperature, pressure, sliding speed, inertia, ...
- Nominal: friction material type, oil type, grooving type

First Approach: One-Hot-Encoding

Pressure, Speed, Energy, ...	Material Type	Material Type A	Material Type B	Material Type C
	A	1	0	0
	A	1	0	0
	B	0	1	0
	C	0	0	1
	C	0	0	1
	C	0	0	1
	B	0	1	0

Difficulties with One-Hot-Encoding in GP

With one-hot encoding:

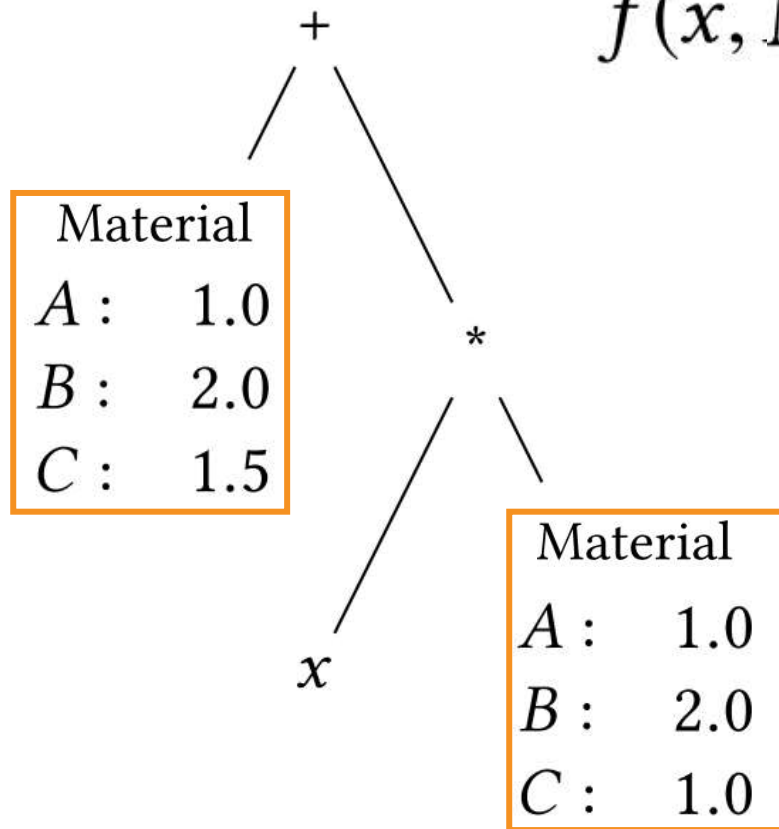
$$\widehat{\Delta T} = T + c_o \text{ cycles} + c_1 p + c_2 \text{Mat}_A p + c_3 \text{Mat}_B p \\ + \frac{1}{p} c_4 \text{Mat}_b + c_5 \text{Mat}_A + \exp(c_6 \text{Mat}_C)$$

With material-dependent “factors-variables”:

$$\widehat{\Delta T} = T + c_o \text{ cycles} + \text{Mat } p + \text{Mat } \frac{1}{p} + \text{Mat}$$

The Novel GP Extension: Factor Variables

$$f(x, \text{Material}) = \underline{\text{Material}} \cdot x + \underline{\text{Material}}$$



x	Material	$f(x, \text{Material})$
3.0	A	4.0
2.0	B	6.0
1.0	C	2.5

Optimization of Factor Variables

We use gradient information and a Quasi-Newton method for optimizing all numeric parameters of symbolic regression models [\[Topchy and Punch, GECCO 2001\]](#) [\[Kommenda et al., GECCO 2013\]](#).

E.g. for optimizing the sum of squared errors:

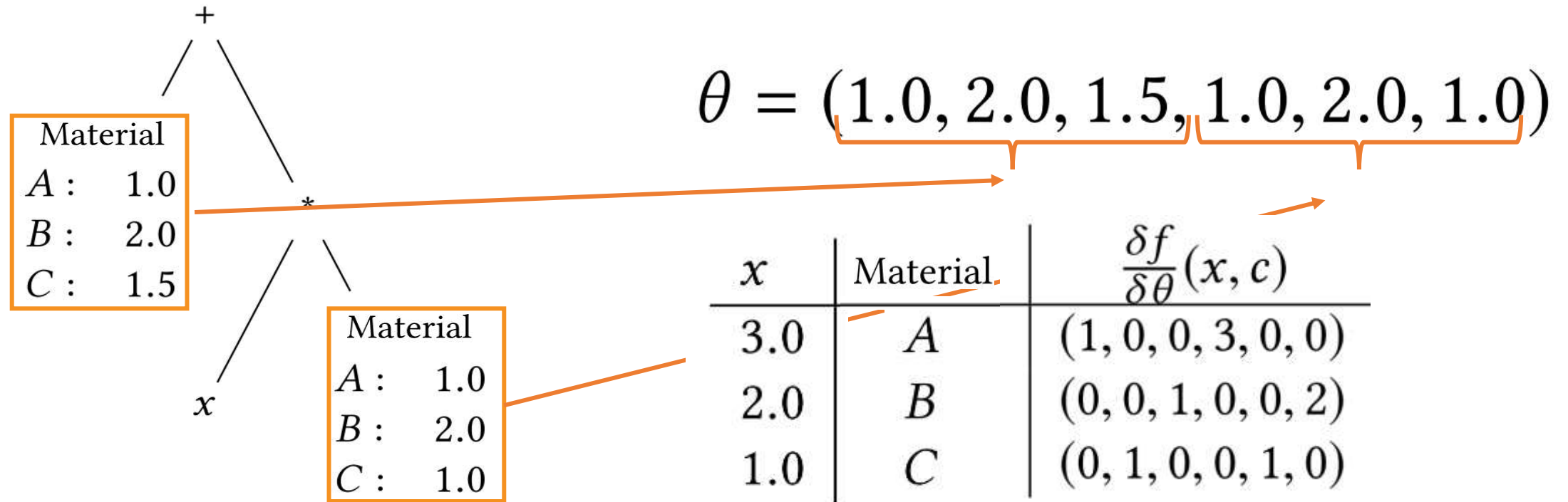
$$\operatorname{argmin}_{\beta} \sum_{i=0}^m (y_i - f(x_i, \beta))^2$$

The gradient of $f(x, \beta)$ over parameters β is useful and can be calculated easily

$$\nabla f = \left(\frac{\partial f}{\partial \beta_1}, \frac{\partial f}{\partial \beta_2}, \dots, \frac{\partial f}{\partial \beta_n} \right)$$

Factor variables simply induce additional numeric parameters!

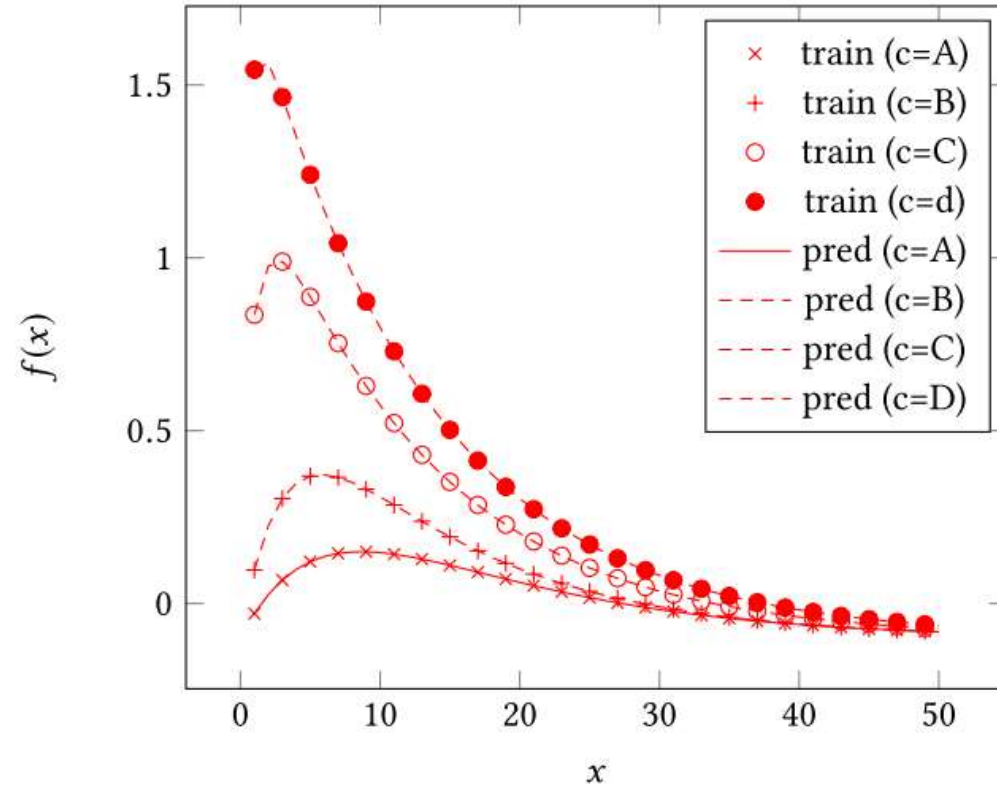
Optimization of Factor Variables



- Gradient can be calculated easily using automatic differentiation with an effort factor of $O(d)$
- No numeric estimate of the gradient!

Does it work?

$$f(x) = \theta_{c,1} \exp(-0.08 \cdot x) - \exp(\theta_{c,2} \cdot x) - 0.1$$



c	θ_1	θ_2
A	1.0	-0.16
B	1.0	-0.32
C	1.5	-0.80
D	2.0	-1.60

It works!

Symbolic Regression Result

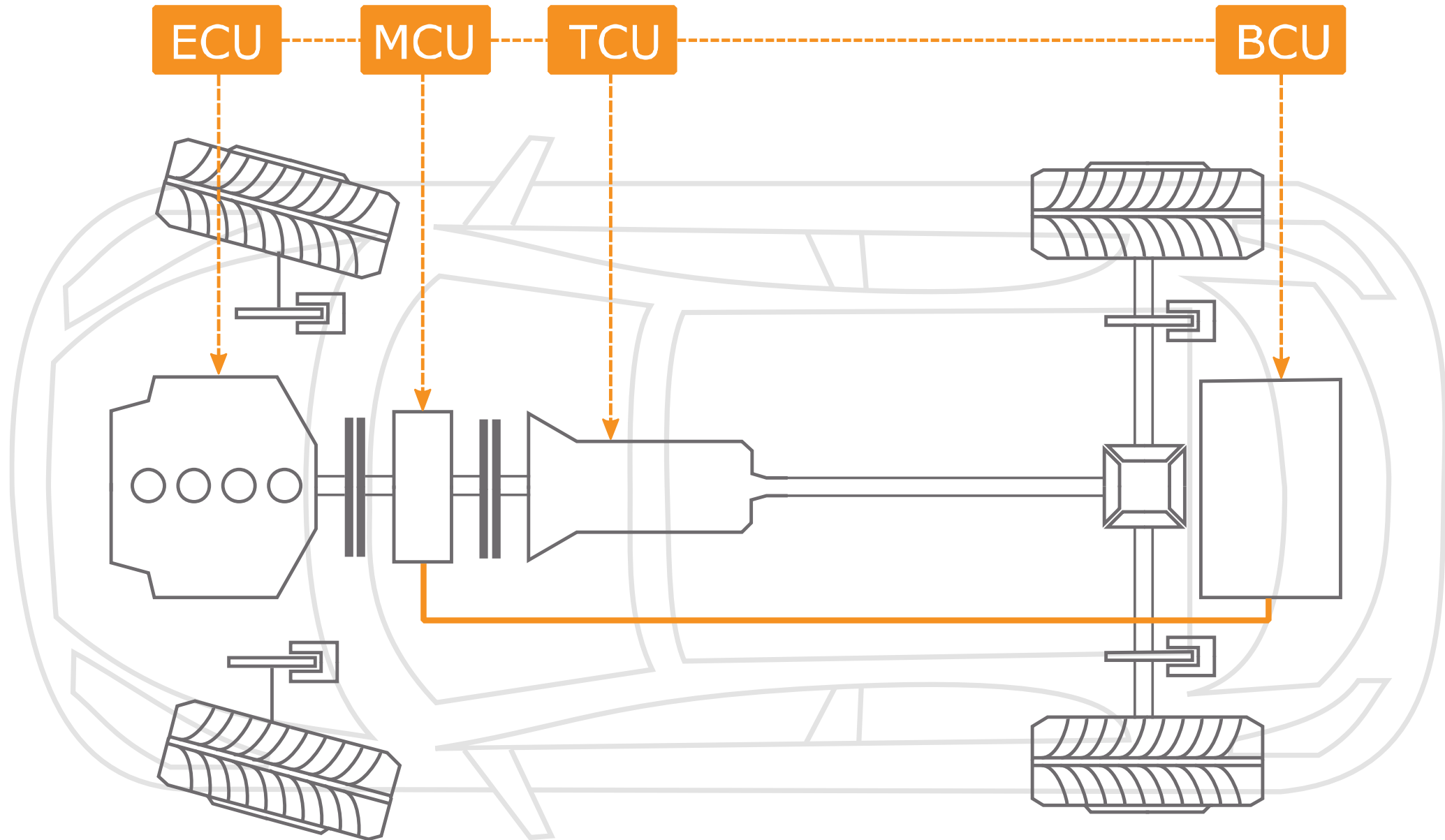
$$\hat{f}(x) = c_0 + c_1 \cdot \exp(x \cdot c_2) + c_3 \cdot \exp(c_4 \cdot x) \cdot \exp(c_5 \cdot x) + c_6$$

c_0	-0.25744	$c_{3,c=A}$	1.0
c_1	-1.0	$c_{3,c=B}$	1.0
$c_{2,c=A}$	-0.16	$c_{3,c=C}$	1.5
$c_{2,c=B}$	-0.32	$c_{3,c=D}$	2.0
$c_{2,c=C}$	-0.8	c_4	-0.034663
$c_{2,c=D}$	-1.6	c_5	-0.045337
		c_6	0.15744

Generating Function

$$f(x) = \theta_{c,1} \exp(-0.08 \cdot x) - \exp(\theta_{c,2} \cdot x) - 0.1$$

c	θ_1	θ_2
A	1.0	-0.16
B	1.0	-0.32
C	1.5	-0.80
D	2.0	-1.60





Using robust generalized fuzzy modeling and enhanced symbolic regression to model tribological systems

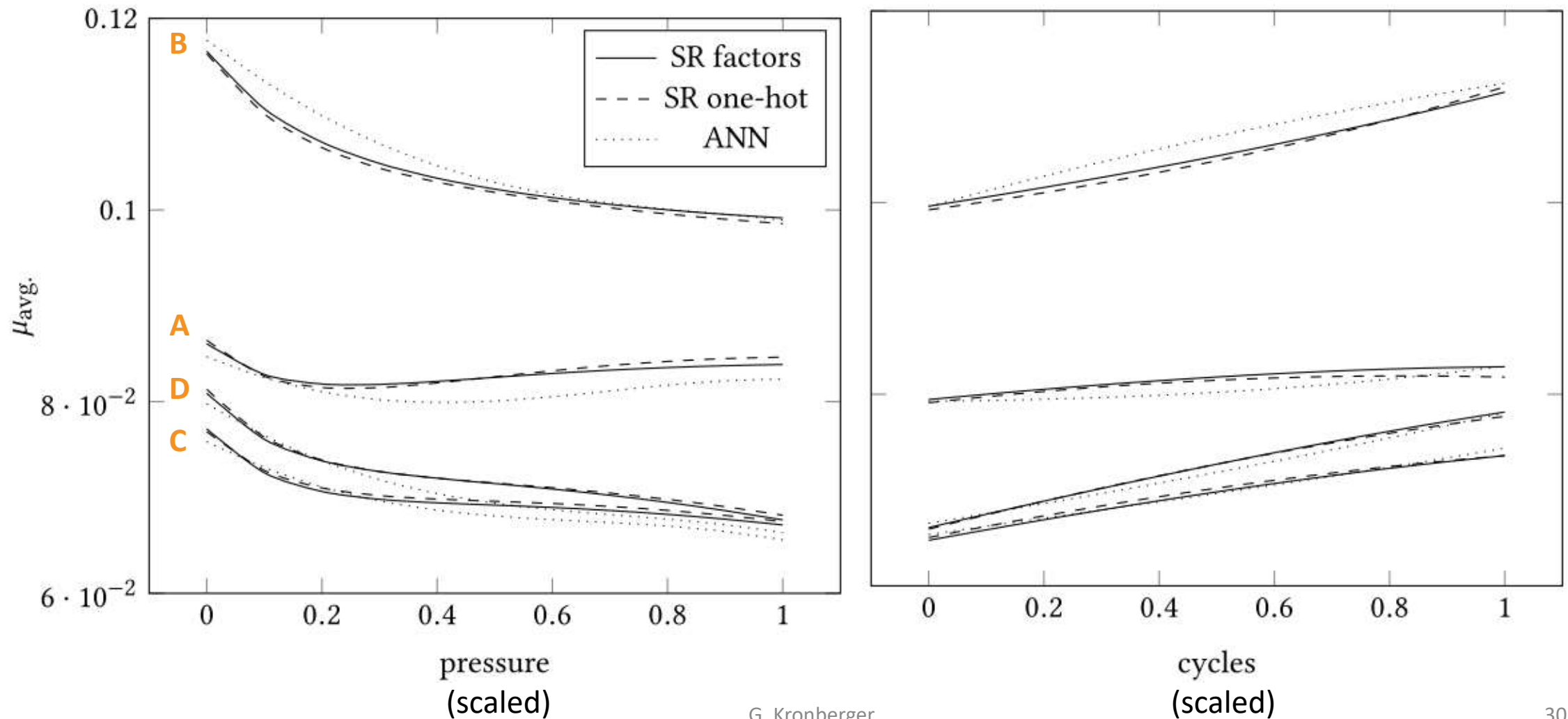
Gabriel Kronberger ^a  , Michael Kommenda ^a , Edwin Lughofer ^b , Susanne Saminger-Platz ^b ,
Andreas Promberger ^c , Falk Nickel ^c , Stephan Winkler ^a , Michael Affenzeller ^a 

Public data set: <http://dev.heuristiclab.com/AdditionalMaterial>

Results for Predicting the Friction Coefficient

Model	average relative error
Linear regression	4.15 %
Artificial neural network	2.86 %
Random forest	3.08 %
Sym. reg. with one-hot-encoding	2.77 %
Sym. reg. with factor variables	2.84 %

Results for Friction Performance Prediction



Results for Friction Performance Prediction

SR with one-hot encoding

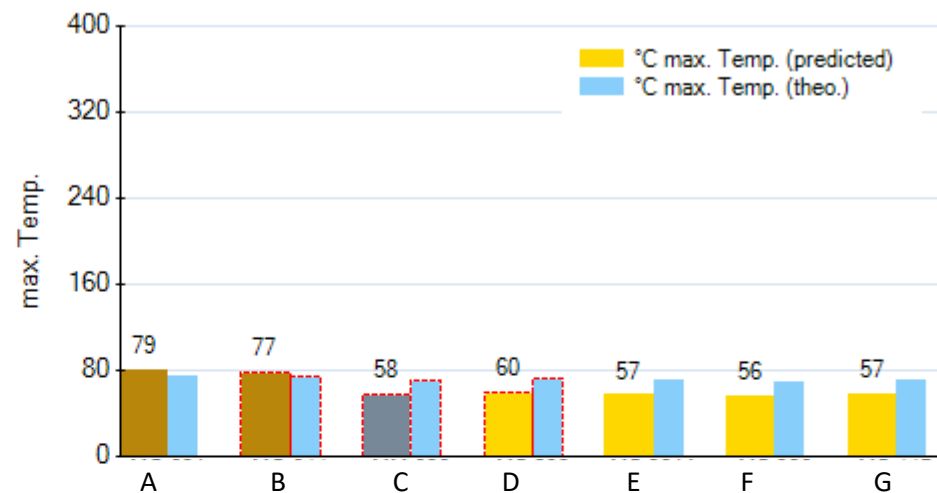
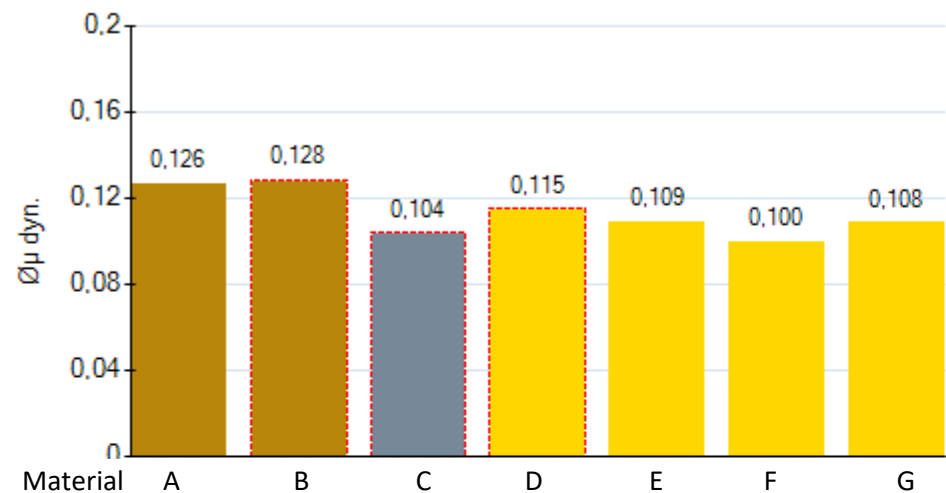
$$\begin{aligned} \widehat{\mu_{avg}} = & \frac{c_0 \cdot \text{cycles} + c_1 \cdot p + c_2 \cdot \underline{M=b} + \exp(c_3 \cdot \underline{M=c}) \cdot c_4}{\exp(c_5 \cdot \underline{M=d} + c_6 \cdot \text{cycles} + c_7 \cdot p) \cdot c_8} \\ & + \exp(c_9 \cdot \underline{M=d} + c_{10} \cdot p) \cdot c_{11} \\ & + \frac{c_{12} \cdot \text{cycles} + c_{13} \cdot \underline{M=b} + c_{14} \cdot p + c_{15} \cdot \underline{M=c} + c_{16} \cdot \underline{M=d}}{\exp(c_{17} \cdot \underline{M=b} + c_{18} \cdot p) \cdot (c_{19} \cdot p + c_{20} \cdot \text{cycles})} + c_{22} \end{aligned}$$

SR with factor variables

$$\begin{aligned} \widehat{\mu_{avg}} = & (c_0 \cdot \text{cycles} + \underline{c_{1,M}} \cdot \exp(c_2 \cdot \text{cycles} + c_3 \cdot p) \\ & + \underline{c_{4,M}} \cdot (c_5 \cdot p + \exp(c_6 \cdot p) + c_7)) \\ & \cdot \frac{1}{c_8 \cdot p + \underline{c_{9,M}} + c_{10}} + c_{11} \end{aligned}$$

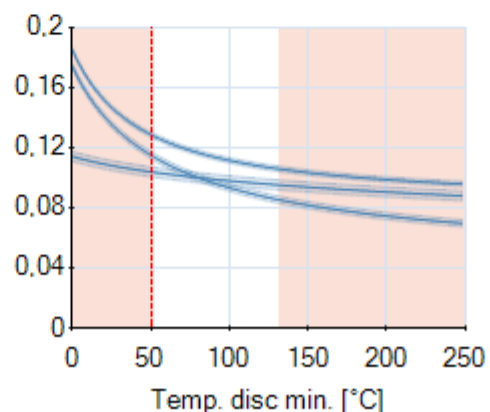
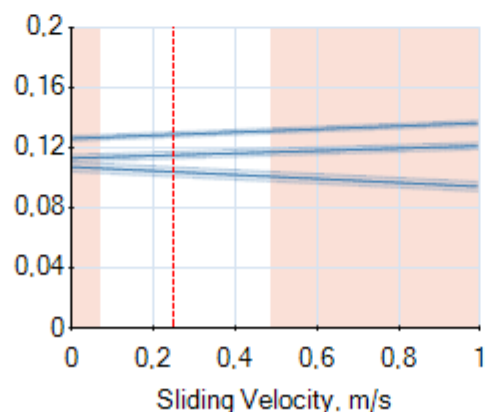
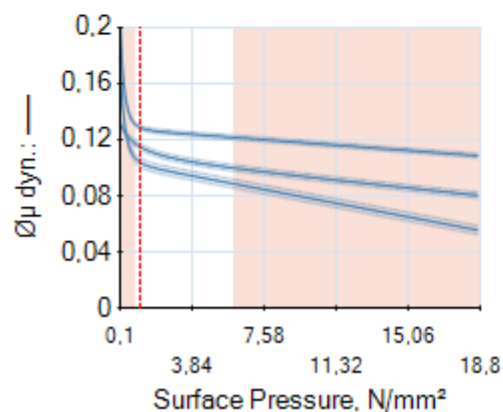
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Parameter: P: — [W/mm²] p: 1,17 [N/mm²] v: 0,3 [m/s] E: — [J/mm²] Tdisc_min: 50 [°C] t: 4,2 [s]



Details

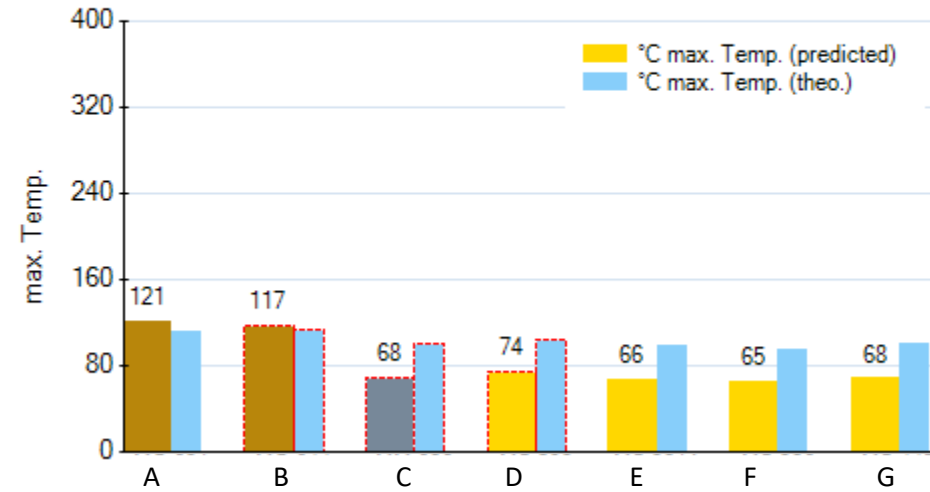
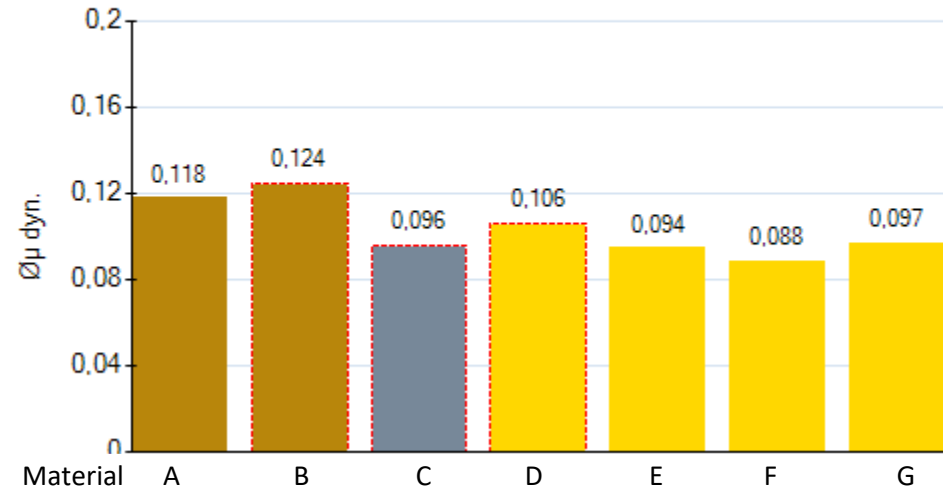
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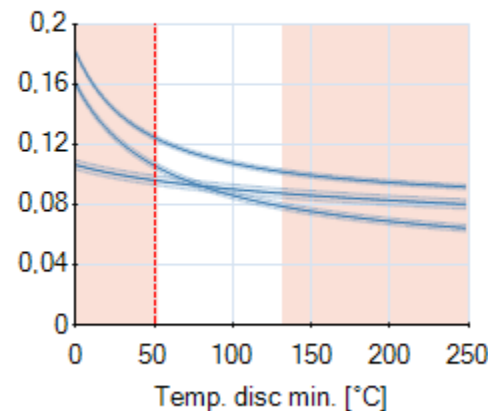
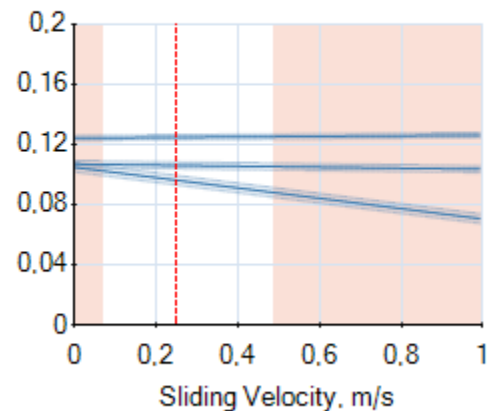
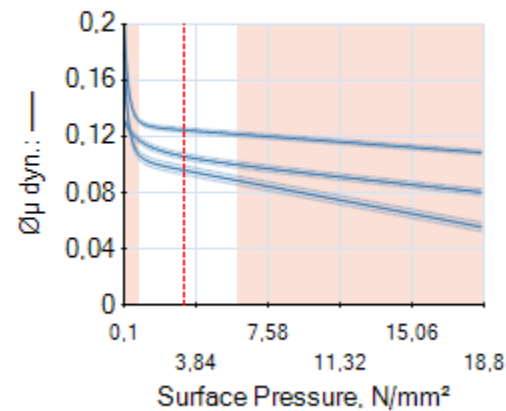
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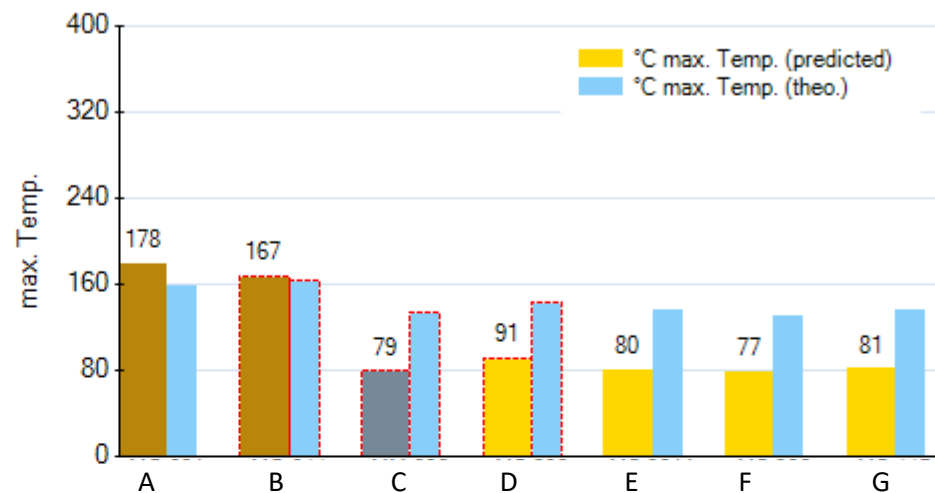
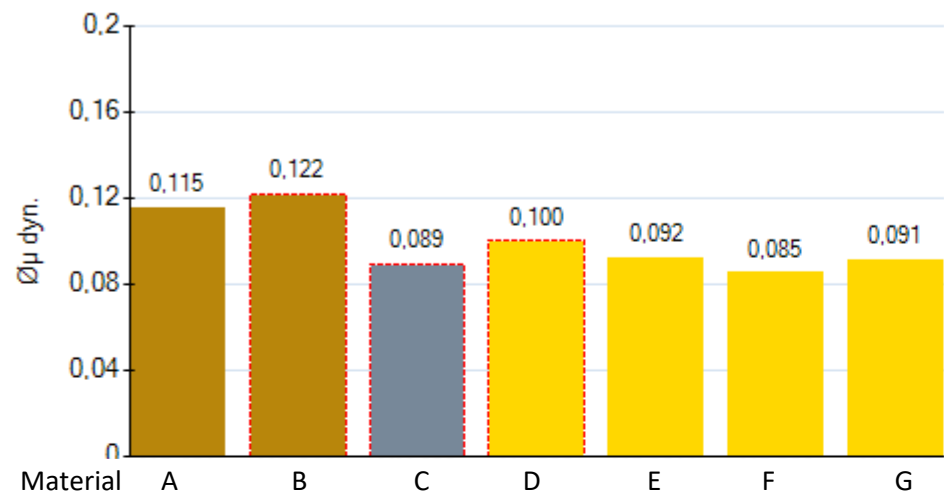
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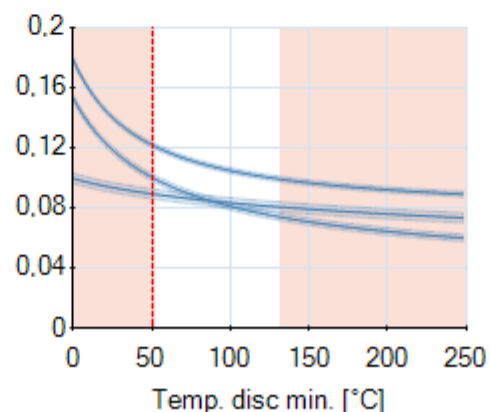
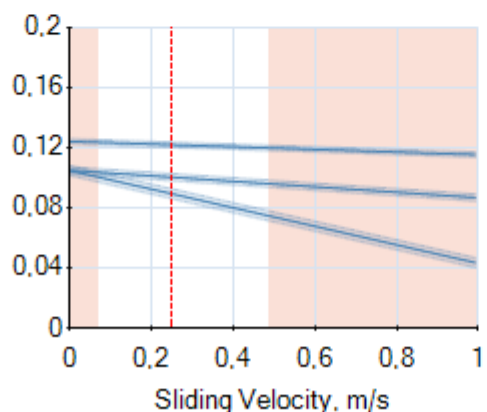
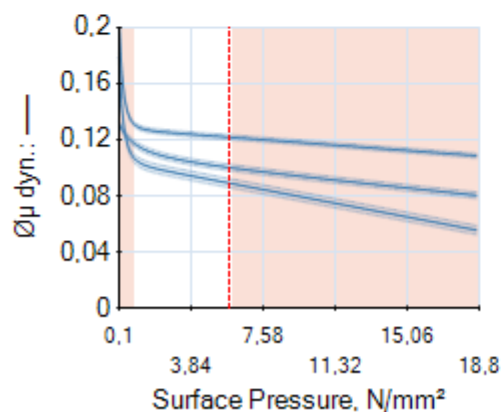
Oil: A

Parameter: P: — [W/mm²] p: 5,84 [N/mm²] v: 0,3 [m/s] E: — [J/mm²] Tdisc_min: 50 [°C] t: 4,2 [s]



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Summary

- Introduced factor variables for genetic programming
- Described gradient-based optimization of factor variables
- Successfully applied to friction system prediction

Factor variables are useful for modeling systems with nominal variables which change the response of the system but not the fundamental mechanics of the system.

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